The Receiving Antenna*

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Summary—After a discussion of the general problem of coupled antennas, the distribution of current in a center-loaded receiving antenna with its axis in the plane of a linearly polarized electric field is derived. Distribution curves are shown for a wide range of lengths, thicknesses, and load impedances. A simple "equivalent" circuit is obtained for the loaded antenna for determining the current in the load. The effective length for a receiving antenna is defined and curves are shown for a wide range of lengths and several thicknesses. An expression for the power transferred to a matched load is derived and curves for computing it are given for antennas over the same range of lengths and thicknesses. Optimun conditions are discussed.

GENERAL INTRODUCTION

IN A recent paper,[1] the distribution of current along a cylindrical center-driven antenna was analyzed. In the present paper the distribution of current along a loaded receiving antenna which is in the far zone of a transmitter will be considered in detail. This is a relatively simple special case of the general problem of the distribution of current along two coupled antennas which will be formulated first. The formal solution will then be limited by the particular conditions for the receiving antenna, the detailed analysis of the general case being reserved for a later paper.

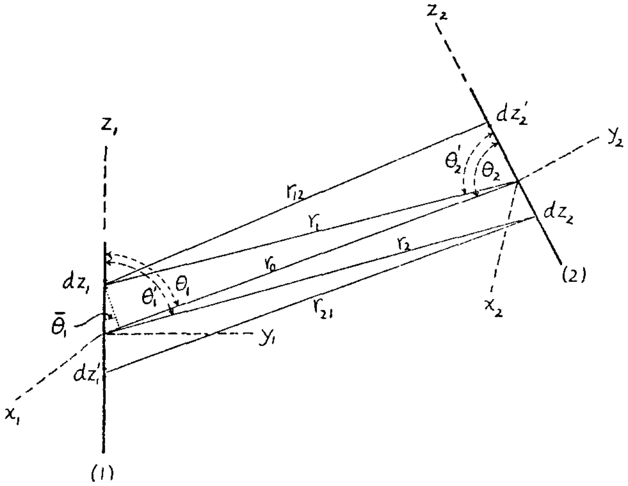

Fig. 1—Orientation of the elements dz' (along the axis) and dz (on the surface) for two antennas.

Consider two straight cylindrical antennas which are oriented in space in an arbitrary way as shown in Fig. 1. Let the origin of a co-ordinate system x_1, y_1, z_1 or R_1, Θ_1, Φ_1 be located at the center of antenna 1; let a second system of co-ordinates x_2, y_2, z_2 or R_2, Θ_2, Φ_2 be located at the center of antenna 2. Whenever convenient the subscript 1 on the co-ordinate system of antenna 1 will be omitted. In each case the z axis coincides with the axes of the antenna. As previously explained[1] the continuity of the tangential component of the electric field along the cylindrical envelope of each antenna leads in this case to

$$E_{1z}^i = E_{1z}^0 \tag{1a}$$

$$E_{2z}^i = E_{2z}^0. \tag{1b}$$

Upon introducing the internal impedance z^i and the total current I for each conductor and making use of the definition of the vector potential one has, just as before,

$$(\partial/\partial z_1)(\text{div } A_1^0) + \beta^2 A_{1z_1}^0 = j(\beta^2/\omega)z_1^i I_1 \tag{2a}$$

$$(\partial/\partial z_2)(\text{div } A_2^0) + \beta^2 A_{2z_2}^0 = j(\beta^2/\omega)z_2^i I_2. \tag{2b}$$

But in the present case A_1^0 is the vector potential on the surface of antenna 1 due to currents not only in antenna 1 but also in antenna 2. Similarly A_2^0 is the vector potential on the surface of antenna 2 due to I_2 and I_1. Thus,

$$A_1^0 = A_{11}^0 + A_{12}^0 \tag{3a}$$

$$A_2^0 = A_{22}^0 + A_{21}^0. \tag{3b}$$

Here A_{11}^0 is the vector potential on the surface of antenna 1 due to I_1; A_{12}^0 is the vector potential on the surface of antenna 1 due to I_2; A_{22}^0 is the vector potential on the surface of antenna 2 due to I_2; A_{21}^0 is the vector potential on the surface of antenna 2 due to I_1. They are given by

$$A_{11}^0 = \frac{\Pi}{4\pi}\int_{-h_1}^{h_1} I_1' \frac{e^{-j\beta r_{11}}}{r_{11}} dz_1' \tag{4a}$$

$$A_{12}^0 = \frac{\Pi}{4\pi}\int_{-h_2}^{h_2} I_2' \frac{e^{-j\beta r_{12}}}{r_{12}} dz_2' \tag{4b}$$

$$A_{22}^0 = \frac{\Pi}{4\pi}\int_{-h_2}^{h_2} I_2' \frac{e^{-j\beta r_{22}}}{r_{22}} dz_1' \tag{4c}$$

$$A_{21}^0 = \frac{\Pi}{4\pi}\int_{-h_1}^{h_1} I_1' \frac{e^{-j\beta r_{21}}}{r_{21}} dz_1'. \tag{4d}$$

The following notation has been used:

$\beta = 2\pi/\lambda$; $\Pi = 4 \times 10^{-7}$ henry per meter

$$r_{11} = \sqrt{(z_1 - z_1')^2 + a_1^2} \tag{5a}$$

$$r_{22} = \sqrt{(z_2 - z_2')^2 + a_2^2} \tag{5b}$$

r_{12} is the distance from dz_1 to dz_2' (5c)

r_{21} is the distance from dz_1' to dz_2. (5d)

a_1 is the radius of antenna 1

a_2 is the radius of antenna 2.

If the two antennas are in the far zone with respect to each other, and r_0 is the distance between their centers so that one can write

$$\beta r_0 \gg 1; \quad r_0 \gg h_1; \quad r_0 \gg h_2, \tag{6}$$

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[1] Ronold King and Charles W. Harrison, Jr., "The distribution of current along a symmetrical center-driven antenna," PROC. I.R.E., vol. 31, pp. 548–567; October, 1943.

(4b) and (4d) reduce to

$$\boldsymbol{A}_{12}{}^0 = \frac{\Pi}{4\pi}\,\frac{e^{-j\beta r_1}}{r_1}\int_{-h_2}^{h_2} I_2{}'e^{j\beta z_2{}'\cos\theta_2{}'}dz_2{}'$$

$$= \frac{\Pi}{4\pi}\,\frac{e^{-j\beta r_1}}{r_1}F(\boldsymbol{I}_2,\,\theta_2). \qquad (7a)$$

$$\boldsymbol{A}_{21}{}^0 = \frac{\Pi}{4\pi}\,\frac{e^{-j\beta r_2}}{r_2}\int_{-h_1}^{h_1} I_1{}'e^{-j\beta z_1{}'\cos\theta_1{}'}dz_1{}'$$

$$= \frac{\Pi}{4\pi}\,\frac{e^{-j\beta r_2}}{r_2}F(\boldsymbol{I}_1,\,\theta_1). \qquad (7b)$$

Here r_1 is the distance from the center of antenna 2 to the element dz_1 in antenna 1 and r_2 is the distance from the center of antenna 1 to an element dz_2 of antenna 2 as shown in Fig. 1. $\theta_1{}'$ is the angle between the z_1 axis and r_2, $\theta_2{}'$ is the angle between the z_2 axis and r_1, θ_1 is the angle between the z_1 axis and r_0, and θ_2 is the angle between the z_2 axis and r_0.

DIFFERENTIAL EQUATION FOR THE UNLOADED RECEIVING ANTENNA

Let antenna 1 be a receiving antenna consisting of a straight cylindrical conductor with no impedance at its center. Let it be in the far zone with respect to a transmitting antenna 2. The equation (2a) applies with $\boldsymbol{A}_{11}{}^0$ given by (4a) and $\boldsymbol{A}_{12}{}^0$ given by (7a).

One can write

$$\boldsymbol{A}_{12}{}^0 = \boldsymbol{K}_{12}e^{-j\beta r_1}. \qquad (8a)$$

From (7a) one has

$$\boldsymbol{K}_{12} = (\Pi/4\pi)F(\boldsymbol{I}_2\theta_2)/r_1). \qquad (8b)$$

$\boldsymbol{K}_{12}$ is practically constant over the length of antenna 1. It has been shown[1] that the vector potential due to the current in an antenna is directed parallel to that antenna except at points very near the small end faces. Accordingly $\boldsymbol{A}_{11}{}^0$ has only the z_1 component $A_{11z}{}^0$. Using this value, and with (8a) substituted in (2a) according to (3a),

$$\{(\partial^2 A_{11z}{}^0/\partial z_1{}^2) + \beta^2 A_{11z}{}^0\} + \{(\partial/\partial z_1)[\mathrm{div}_1\,(\boldsymbol{K}_{12}e^{-j\beta r_1})]$$
$$+ \beta^2\,\boldsymbol{K}_{12z}e^{-j\beta r_1}\} = j(\beta^2/\omega)z_1{}^i\boldsymbol{I}_1. \qquad (9)$$

The operation div_1 involves only the co-ordinates of antenna 1 with respect to which $\boldsymbol{K}_{12}$ is sensibly constant. Accordingly div $\boldsymbol{K}_{12}$ vanishes and, writing s for r_1,

$$\mathrm{div}_1\,[\boldsymbol{K}_{12}e^{-j\beta r_1}] = (\boldsymbol{K}_{12},\,\mathrm{grad}_1\,\mathrm{e}^{-j\beta s})$$
$$= (\boldsymbol{K}_{12},\,\boldsymbol{s}_1)[-j\beta e^{-j\beta s}]. \qquad (10)$$

Since both $\boldsymbol{K}_{12}$ and the unit vector $\boldsymbol{s}_1$, directed along $s=r_1$, are independent of differentiation with respect to z_1,

$$(\partial/\partial z_1)[\mathrm{div}_1\,(\boldsymbol{K}_{12}e^{-j\beta s})] = (\boldsymbol{K}_{12},\,\boldsymbol{s}_1)\{-\beta^2 e^{-j\beta s}\}(\partial s/\partial z_1). \quad (11)$$

With (8a), and noting that

$$(\partial s/\partial z_1) = \sin\bar{\theta}_1, \qquad (12)$$

where $\bar{\theta}_1$ is the complement of θ_1 as shown in Figs. 1 and 2,

$$(\partial/\partial z_1)[\mathrm{div}_1\,(\boldsymbol{K}_{12}e^{-j\beta s})] = -\beta^2 A_{12s}{}^0\sin\bar{\theta}_1. \qquad (13)$$

Here,

$$A_{12s}{}^0 = (\boldsymbol{K}_{12},\,\mathrm{s}_1)e^{-j\beta s} \qquad (14)$$

is the component of $\boldsymbol{A}_{12}{}^0$ parallel to the line $s=r_1$.

The general equation (9) may now be written as follows:

$$(\partial^2 A_{11z}{}^0/\partial z_1{}^2) + \beta^2 A_{11z}{}^0 + \beta^2(A_{12z}{}^0 - A_{12s}{}^0\sin\bar{\theta}_1)$$
$$= j(\beta^2/\omega)z_1{}^i I_1. \qquad (15)$$

Since the distance $r_1=s$ is measured from the center of the transmitting antenna 2 to any element dz_1 along the axis of antenna 1, it is a function of z_1. Thus, for antenna 1 in the far zone with respect to antenna 2 one has (see Fig. 1)

$$r_1 = s = r_0 - z_1\sin\bar{\theta}_1. \qquad (16)$$

Here $\bar{\theta}_1$ is the angle between the axis of antenna 1 and a perpendicular dropped from dz_1 on to r_0 as shown in Figs. 1 and 2.

Thus $$\boldsymbol{A}_{12}{}^0 = \boldsymbol{K}_{12}e^{-j\beta s} = \boldsymbol{K}_{12}e^{-j\beta r_0}e^{+j\beta z_1\sin\bar{\theta}_1}. \qquad (17)$$

This may be rearranged as follows:

$$\boldsymbol{A}_{12}{}^0 = \tfrac{1}{2}[\boldsymbol{A}_{12}{}^0(z)+\boldsymbol{A}_{12}{}^0(-z)]+\tfrac{1}{2}[\boldsymbol{A}_{1z}{}^0(z)-\boldsymbol{A}_{12}{}^0(-z)] \quad (18)$$

with

$$\tfrac{1}{2}[\boldsymbol{A}_{12}{}^0(z) + \boldsymbol{A}_{12}{}^0(-z)] = \boldsymbol{K}_{12}e^{-j\beta r_0}\cos(\beta z_1\sin\bar{\theta}_1) \quad (18a)$$
$$\tfrac{1}{2}[\boldsymbol{A}_{12}{}^0(z) - \boldsymbol{A}_{12}{}^0(-z)] = j\boldsymbol{K}_{12}e^{-j\beta r_0}\sin(\beta z_1\sin\bar{\theta}_1). \quad (18b)$$

The relation (18a) implies the following symmetry relations:

$$A_{12}{}^0(z) = A_{12}{}^0(-z); \qquad I(z) = I(-z). \qquad (19a)$$

The relation (18b) satisfies the symmetry conditions:

$$A_{12}{}^0(z) = -A_{12}{}^0(-z); \quad I(z) = -I(-z). \quad (19b)$$

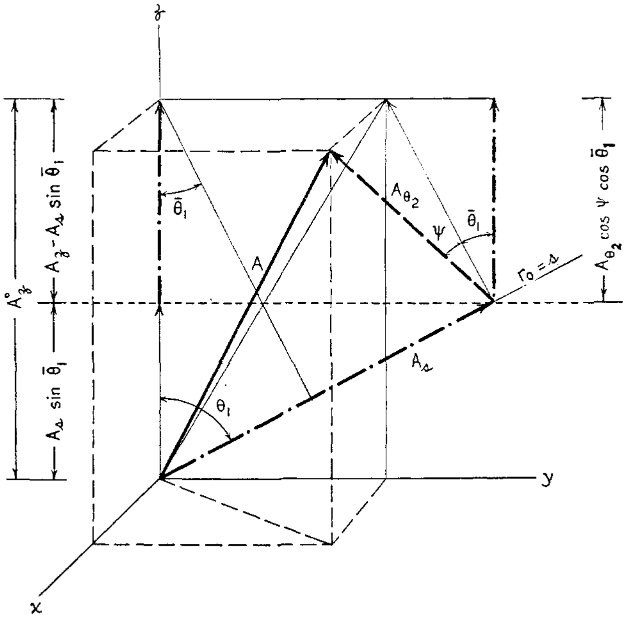

Fig. 2—The component of the vector potential $\boldsymbol{A}$ due to the current in a distant antenna (2) along antenna 1 when this is placed along the z axis shown in the figure.

Each of the two parts in (18), as written, respectively, in (18a) and (18b), leads to an independent solution for the current. The general solution is the superposition of the two. It is to be noted that the distribution of current that is obtained using (18a) and which obeys the symmetry relation for current in (19a) is one in which the current is unidirectional through the center of the antenna. If a load is placed at the center a potential

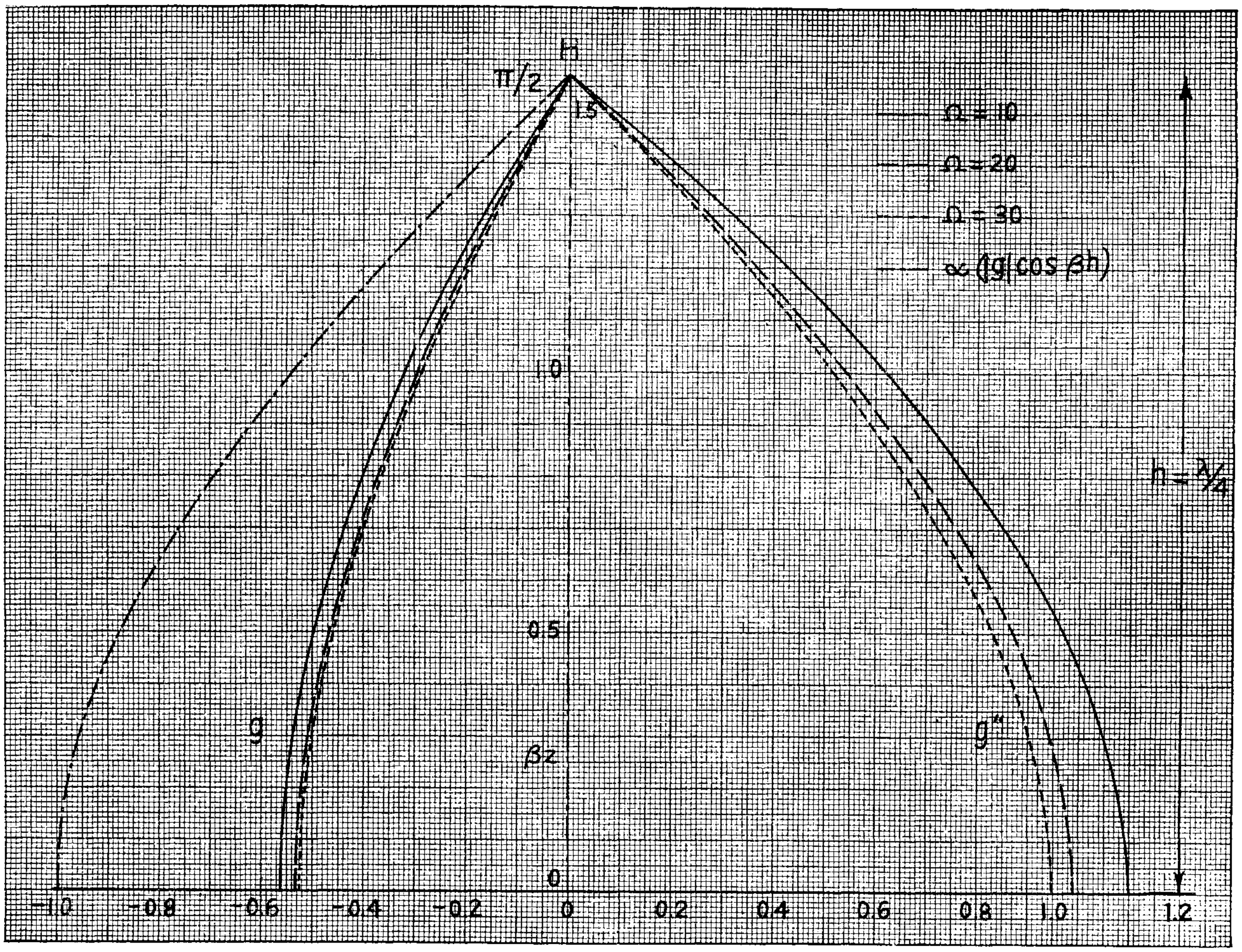

Fig. 3—The functions g' and g'' for an unloaded receiving antenna of half length $h=\lambda/4$. g'' is proportional to the component of current in phase with the electric field, g' is proportional to the component in phase quadrature with the field if this is parallel to the antenna.

difference can be established across it. On the other hand the distribution that is obtained from (18b) and which obeys (19b) is one in which the currents from the two halves flow simultaneously toward or away from the center reducing to zero at the center. Such a current can establish no potential difference across a load which is symmetrically placed at the center. If the load is not placed at the center of a symmetrical antenna the current due to (18b) is also significant in maintaining a potential difference across it. When the load is introduced in the following analysis it will be assumed that it is placed at the center of the antenna so that only (18a) is of importance in determining the current in it and the power transferred to it. Accordingly from the point of view of this load at the center one may write (18a) for A_{12}^0 in (15). For convenience in writing (18a) will be denoted by A_{12}^0. Thus for the present analysis let

$$\begin{aligned} A_{12}^0 &= \tfrac{1}{2}\left[A_{12}^0(z_1) + A_{12}^0(-z_1)\right] \\ &= K_{12}e^{-j\beta r_0}\cos(\beta z_1 \sin\theta_1), \qquad (20) \end{aligned}$$

with the understanding that (18b) has been omitted. It is important to note, however, that the distribution of current along the antenna obtained using (20) is, in general, not the entire current at other points than the center. Distribution curves computed below give only the components of current due to (18a), but since they are computed only for the special case for which $\sin\bar{\theta}_1 = 0$, so that (18b) vanishes identically, they actually represent the entire current. Upon substituting (20) in (15) with the notation

$$q = \beta\sin\bar{\theta}_1 \qquad (21)$$

one has

$$\begin{aligned} &(\partial^2 A_{11z}/\partial z_1^2) + \beta^2 A_{11z} \\ &\quad = (j\beta^2/\omega)z_1^i I_{1z} - \beta^2 e^{-j\beta r_0}\cos(qz_1)\left[K_{12z} - K_{12s}\sin\bar{\theta}_1\right]. \qquad (22) \end{aligned}$$

Let the following shorthand notation be introduced with (17):

$$U \equiv -\left(j\omega(A_{12z}^0 - A_{12s}^0\sin\bar{\theta}_1)/\beta\cos^2\bar{\theta}_1\right)_{z_1=0} \qquad (23a)$$

or $U \equiv -j\omega(K_{12z} - K_{12s}\sin\bar{\theta}_1)e^{-j\beta r_0}/\beta\cos^2\bar{\theta}_1. \qquad (23b)$

The components of the vector potential in (23) *are taken at* $z_1 = 0$, *so that the function* U *is referred to the center of the antenna.* Here U is dimensionally a potential difference measured in volts. With this notation and (21), (22) may be written as follows:

$$\begin{aligned} &(\partial^2 A_{11z}/\partial z_1^2) + \beta^2 A_{11z} \\ &\quad = j(\beta^2/\omega)\left[I_{1z}z^i - \beta U(1-(q^2/\beta^2))\cos qz_1\right]. \qquad (24) \end{aligned}$$

A formally similar nonhomogeneous wave equation was derived by Hallén[2] following a somewhat different method.

The function U defined by (23) may be expressed in a

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[2] E. Hallén, "Theoretical investigations into the transmitting and receiving qualities of antennas," *Nova Acta*, Royal Soc. Sciences, (Uppsala) vol. 11, pp. 1–44; November, 1938.

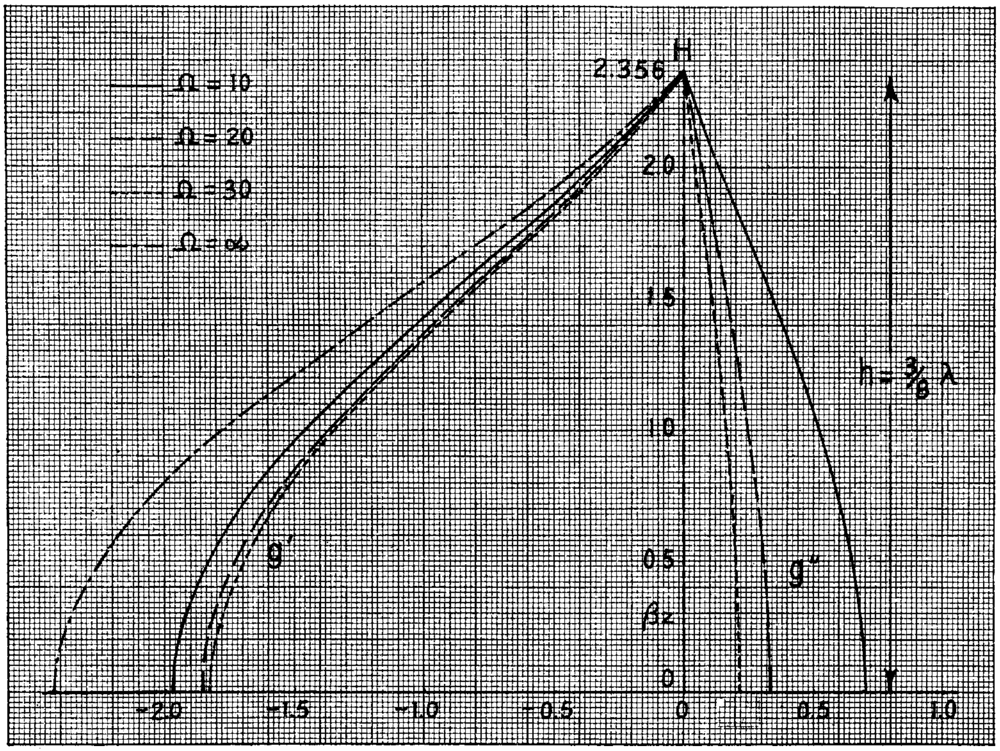

Fig. 4—The functions g' and g'' for an unloaded receiving antenna of half-length $h=3\lambda\backslash 8$.

different way. Since the receiving antenna 1 is in the far zone of the transmitting antenna 2 the entire far-zone electric field may be derived from (7a) according to

$$E_{12\theta_2} = -j\omega A_{12\theta_2}$$
$$= \frac{j\omega\Pi}{4\pi}\,\frac{e^{-j\beta r_1}}{r_1}\int_{-h^2}^{h_2} I_2'^{\,j\beta z_2'\cos\theta_2'}\sin\theta_2' dz_2'. \qquad (25)$$

Here the components $E_{12\theta_2}$ and $A_{12\theta_2}$ are directed tangent to a great sphere about the transmitting antenna 2 along the spherical co-ordinate θ_2 with origin at the center of antenna 2. They are necessarily perpendicular to a radius vector drawn from this origin in any direction, in particular, perpendicular to a line $r_1=s$ drawn to any point on antenna 1. Because s is large all such lines are practically parallel to each other and to r_0. The component of A_{12} perpendicular to s is shown in Fig. 2. (The subscripts 12 on $A_{12\theta_2}$ have been omitted in Fig. 2. They will be omitted below.) It is clear from the figure that

$$(A_{12z}^0 - A_{12s}^0 \sin\bar{\theta}_1)_{z_1=0} = (A_{\theta_2}^0 \cos\psi\cos\bar{\theta}_1)_{z_1=0}. \qquad (26)$$

Accordingly, with (25) and (23a),

$$U = -j\omega(A_{\theta_2}^0\cos\psi/\beta\cos\bar{\theta}_1)_{z_1=0}$$
$$= (E_{\theta_2}\cos\psi/\beta\cos\bar{\theta}_1)_{z_1=0}. \qquad (27)$$

(The subscripts 12 have been omitted from $E_{12\theta_2}$.) This is the form introduced by Hallén. It is often more convenient than (23) because (25) is usually more readily evaluated, and also because E_{θ_2} may be measured experimentally.

It is important to note that when the receiving antenna 1 lies in a plane perpendicular to the line r_0 joining its center to the distant transmitter that it is also perpendicular to $r_1=s$, so that, from Fig. 2,

$$\bar{\theta}_1 = 0; \qquad \cos\bar{\theta}_1 = 1 \qquad (28)$$

and from (21) and (27),

$$q = 0; \qquad U = (E_{\theta_2}\cos\psi/\beta)_{z_1=0}. \qquad (29)$$

FORMAL SOLUTION OF THE DIFFERENTIAL EQUATION

The nonhomogeneous wave equation (24) may be solved for A_{11z}^0 in the same way as has been shown.[1] Omitting the subscript 1 on z_1 throughout, the complementary function is exactly as before.

$$A_c^0 = -j/c[C_1\cos\beta z + C_2\sin\beta|z|] \qquad (30)$$

after taking into account the symmetry condition (18). The particular integral includes a term in U in addition to that previously given with z^i as a factor. It is

$$A_p^0 = \frac{j}{c}\left[z^i\int_0^z I(s)\sin\beta(z-s)ds - U\cos qz\right]. \qquad (31)$$

It is readily verified by direct differentiation that (31) satisfies (24). Accordingly the general solution of (24) is

$$A_{11z}^0 = \frac{-j}{c}\left\{C_1\cos\beta z + C_2\sin\beta|z|\right.$$
$$\left. - z^i\int_0^z I(s)\sin\beta(z-s)ds + U\cos qz\right\}. \qquad (32)$$

If the antenna were driven at the center by a driving potential V_0^e, the arbitrary constant C_2 would have the value $\frac{1}{2}V_0^e$. Since V_0^e is zero for the unloaded receiving antenna, C_2 vanishes, and the solution simplifies to

$$A_{11z}^0 = \frac{-j}{c}\left\{C_1\cos\beta z\right.$$
$$\left. - z^i\int_0^z I(s)\sin\beta(z-s)ds + U\cos qz\right\}. \qquad (33)$$

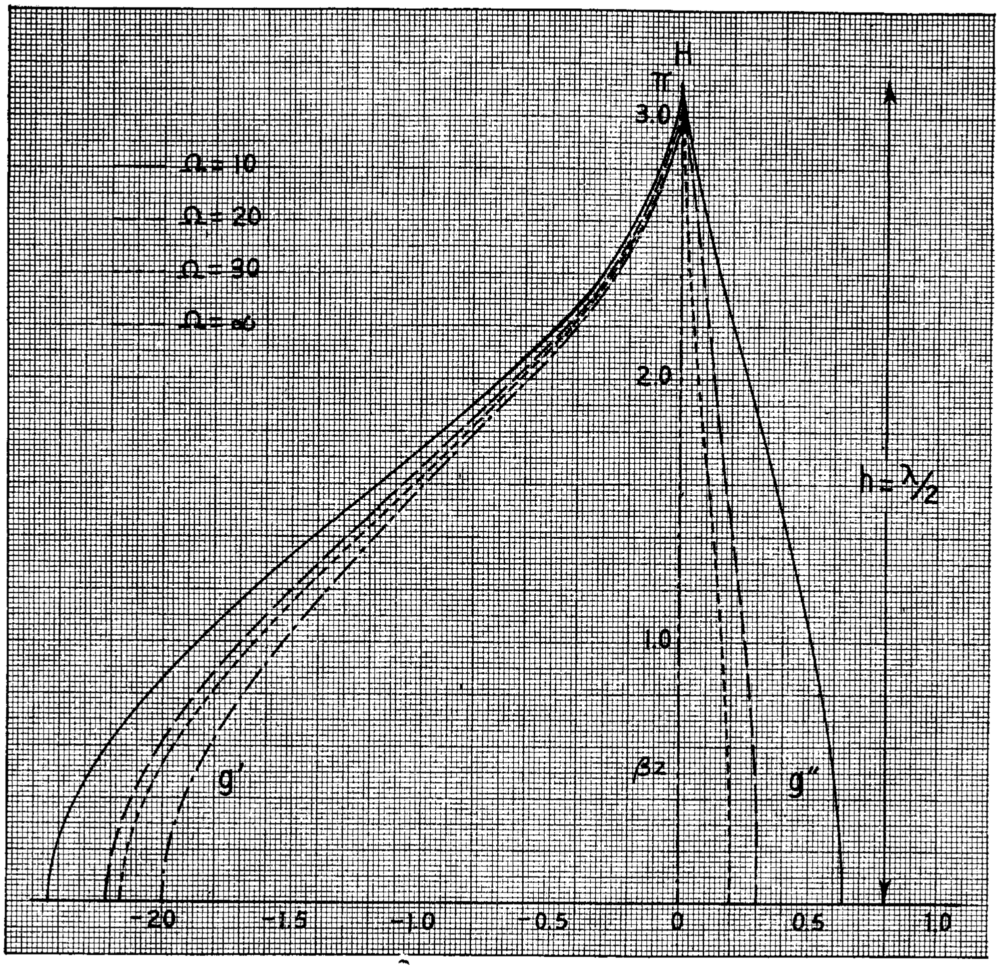

Fig. 5—The functions g' and g'' for an unloaded receiving antenna of half-length $h=\lambda/2$.

Upon introducing the integral (4a) for $A_{1\text{I}z}{}^0$ in (33) and expanding this exactly as before,[1] one obtains an expression for I_{1z} like (35) of reference 1 but with $U \cos qz$ appearing in place of $\frac{1}{2}V_0{}^e \sin \beta|z|$. The value of I_{1z} vanishes at the ends where $z=h$. By setting $z=h$ one obtains an expression like (36) in reference 1 but with $U \cos qh$ appearing instead of $\frac{1}{2}V_0{}^e \sin \beta h$. The final solution for I_z is exactly like (38), reference 1, with $U(\cos qh \cos \beta z - \cos qz \cos \beta h)$ written instead of $\frac{1}{2}V_0{}^e(\sin \beta|z| - \sin \beta h)$. The same result was obtained by Hallén.

By applying exactly the same method of successive approximations in powers of $1/\Omega$ where, as before,

$$\Omega = 2 \ln (2h/a) \tag{34}$$

with a the radius and $2h$ the length of the antenna, one obtains the following formula for I_z with all terms involving higher powers of $1/\Omega$ assumed negligible:

$$I_z = \frac{j4\pi U}{\Omega R_c}\left\{\frac{[\cos qh \cos \beta z - \cos qz \cos \beta h] + (1/\Omega)[m_1{}^{\text{I}} + jm_1{}^{\text{II}}]}{\cos \beta h + (1/\Omega)[A_1{}^{\text{I}} + jA_1{}^{\text{II}}]}\right\}. \tag{35}$$

Here $R_c \equiv \sqrt{\Pi/\Delta} = 376.7$ ohms. Just as before,

$$A_1{}^{\text{I}} + jA_1{}^{\text{II}} = F_1(h) \tag{36}$$

while,

$$m_1{}^{\text{I}} + jm_1{}^{\text{II}} = F_1(z) \cos qh - F_1(h) \cos qz + S_1(h) \cos \beta z - S_1(z) \cos \beta h. \tag{37}$$

The functions $S_1(z)$ and $S_1(h)$ are given in the Appendix. The functions $F_1(z)$ and $F_1(h)$ may be found in Appendix II of reference 1. For convenience let the numerator in (35) be denoted by

$$n^{\text{I}} + jn^{\text{II}} = ne^{j\psi_n} \tag{38}$$

with $n^{\text{I}} = \cos \beta z - \cos \beta h + m_1{}^{\text{I}}/\Omega$; $n^{\text{II}} = m_1{}^{\text{II}}/\Omega$; (39)

and $n = \sqrt{(n^{\text{I}})^2 + (n^{\text{II}})^2}$; $\psi_n = \tan^{-1}(n^{\text{II}}/n^{\text{I}})$. (40)

Similarly let the denominator in (35) be denoted

$$D^{\text{I}} + jD^{\text{II}} = De^{j\psi_D} \tag{41}$$

with $D^{\text{I}} = \cos \beta h + A_1{}^{\text{I}}/\Omega$; $D^{\text{II}} = A_1{}^{\text{II}}/\Omega$ (42)

and $D = \sqrt{(D^{\text{I}})^2 + (D^{\text{II}})^2}$; $\psi_D = \tan^{-1}(D^{\text{II}}/D^{\text{I}})$. (43)

Also let $g' = n \cos (\psi_D - \psi_n)$ (44)

$$g'' = n \sin (\psi_D - \psi_n). \tag{45}$$

Here g' and g'' are functions of z, whereas D is not. With this notation (35) becomes

$$I_z = (4\pi U/\Omega R_c D)(g'' + jg') = (U/30\Omega D)(g'' + jg'). \tag{46}$$

Thus g'' is proportional to the component of current in phase with U and hence with the electric field at the center of the antenna; g' is proportional to the component of current in phase quadrature with U and

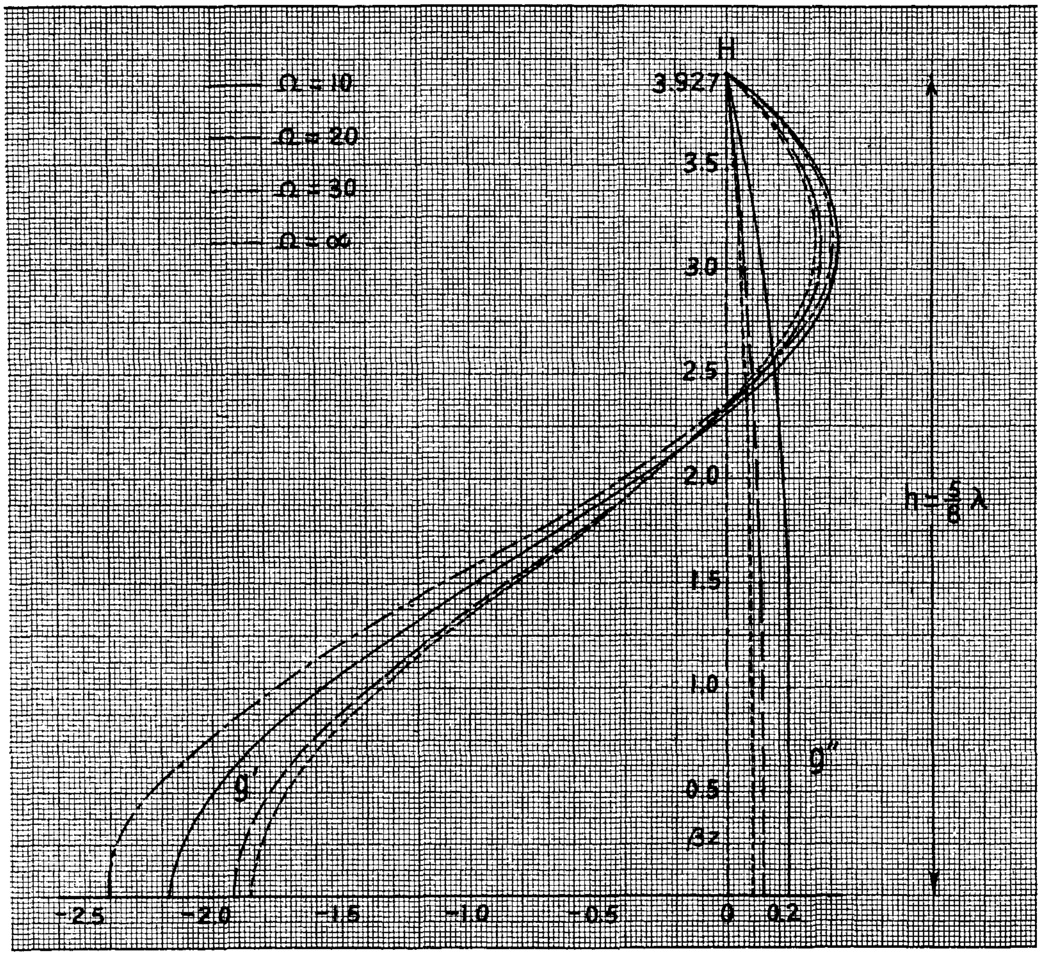

Fig. 6—The functions g' and g'' for an unloaded receiving antenna of half-length $h=5\lambda/8$.

the field. In amplitude-phase-angle form one has

$$I_z = (U/30\Omega D) \mid g \mid e^{j\theta} \qquad (47)$$

$$\mid g \mid = \sqrt{(g'')^2 + (g')^2}; \quad \theta = \tan^{-1} (g'/g''). \qquad (48)$$

Numerical Solution for a Receiving Antenna in a Plane of Constant Phase of a Linearly Polarized Field

In carrying out numerical computations using (46) and (47) only the important special case of a receiving antenna in the plane of constant phase of a linearly polarized electric field will be considered.[3] This case is defined by (28) and the simplifying consequences are contained in (29). Subject to (28) the complicated function given in (37) reduces to the relatively very simple form,

$$m_1{}^{\mathrm{I}} + jm_2{}^{\mathrm{II}} = F_1(z) - F_1(h) \qquad (49)$$

Curves showing the functions g'' and g' for use with formula (46) have been computed subject to (28). They are shown in Figs. 3 to 6 for the same lengths and thicknesses previously calculated for the driven antenna. (Reference 1, Figs. 7 to 10.) Curves giving $\sqrt{(g'')^2+(g')^2}$ and $\theta=\tan^{-1}(g'/g'')$ also subject to (28) are given in Figs. 7 to 10. Values of the factor $1/30\Omega D$ for the several cases are shown in Table I. Figs. 3 to 10 with Table I completely characterize the distribution of current along a highly conducting, straight, cylindrical antenna of radius a and half-length h placed in the radiation zone of a transmitting antenna and oriented so that it lies in a plane perpendicular to the line joining it to the transmitter; i.e., the antenna must lie in a plane of constant phase of a linearly polarized electric field. A comparison of these figures with the corresponding ones for a driven antenna as given[1] reveals that *the distributions of current in the two cases are completely different.* If account is taken of the factors $1/30\Omega D$ or $1/60\Omega D$ the amplitude of the current for a constant applied voltage in the driven case and for a given electric field in the receiving case is seen to increase as the ratio

TABLE I

$\frac{h^*}{\lambda}$	$H=\beta h$ (rad)	$\frac{1}{30\Omega D}$		$\frac{1}{60\pi\Omega D}$	
		$\Omega=10$	$\Omega=30$	$\Omega=10$	$\Omega=30$
0.25	1.571	23.64×10^{-3}	23.62×10^{-3}	7.54×10^{-3}	7.53×10^{-3}
0.375	2.356	4.64	1.57	1.48	0.50
0.50	3.141	3.64	1.18	1.16	0.376
0.625	3.927	5.61	1.66	1.79	0.529

* h is the half-length of a symmetrical center-loaded antenna; a is its radius; $\Omega \cong 2 \ln (2h/a)$.

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[3] Important aspects of the more general case are described in a companion to this paper entitled, "The receiving antenna in plane-polarized field of arbitrary orientation," Proc. I.R.E., this issue, pp. 35–50.

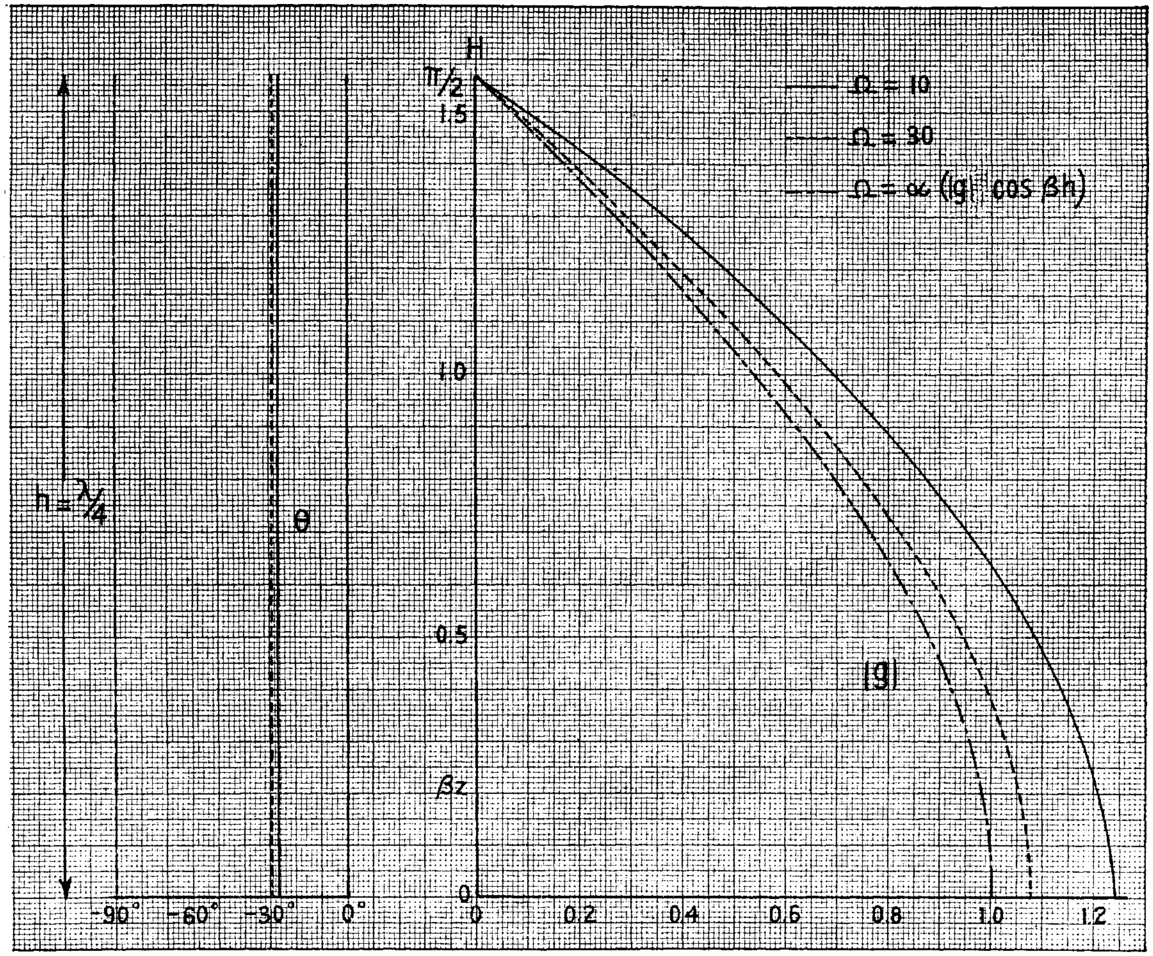

Fig. 7—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $=\lambda/4$; $|g|$ is proportional to the magnitude of the current; θ is its angle referred to the electric field.

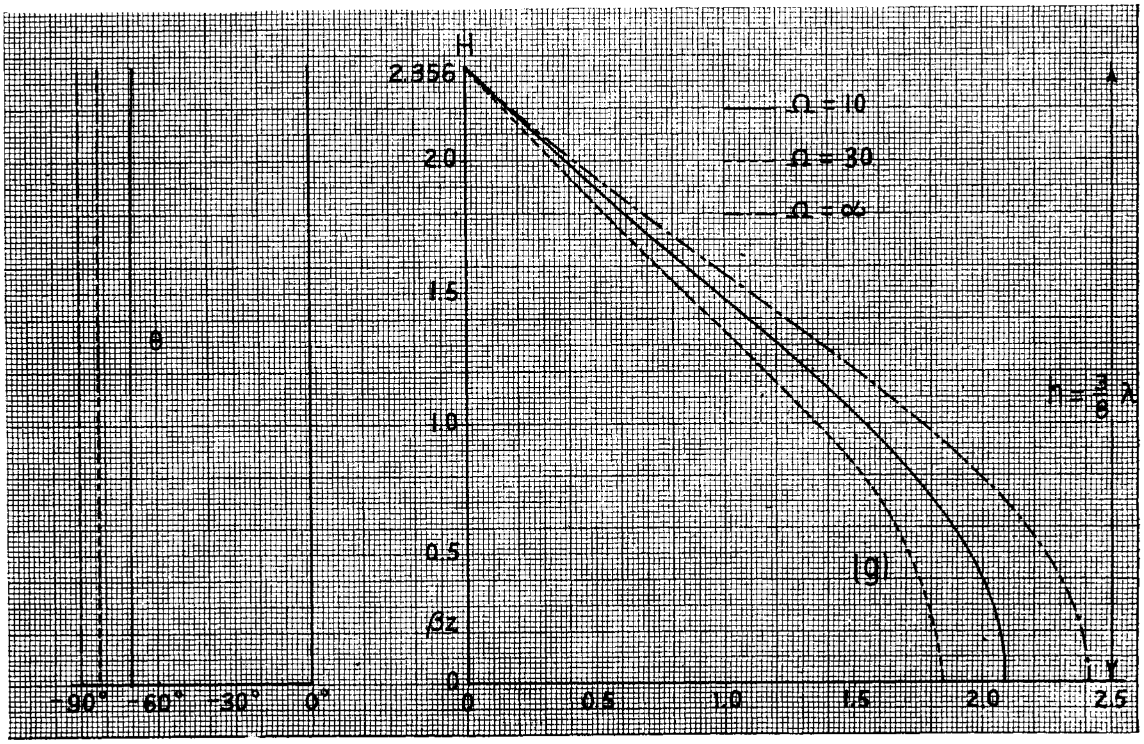

Fig. 8—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $h=3\lambda/8$.

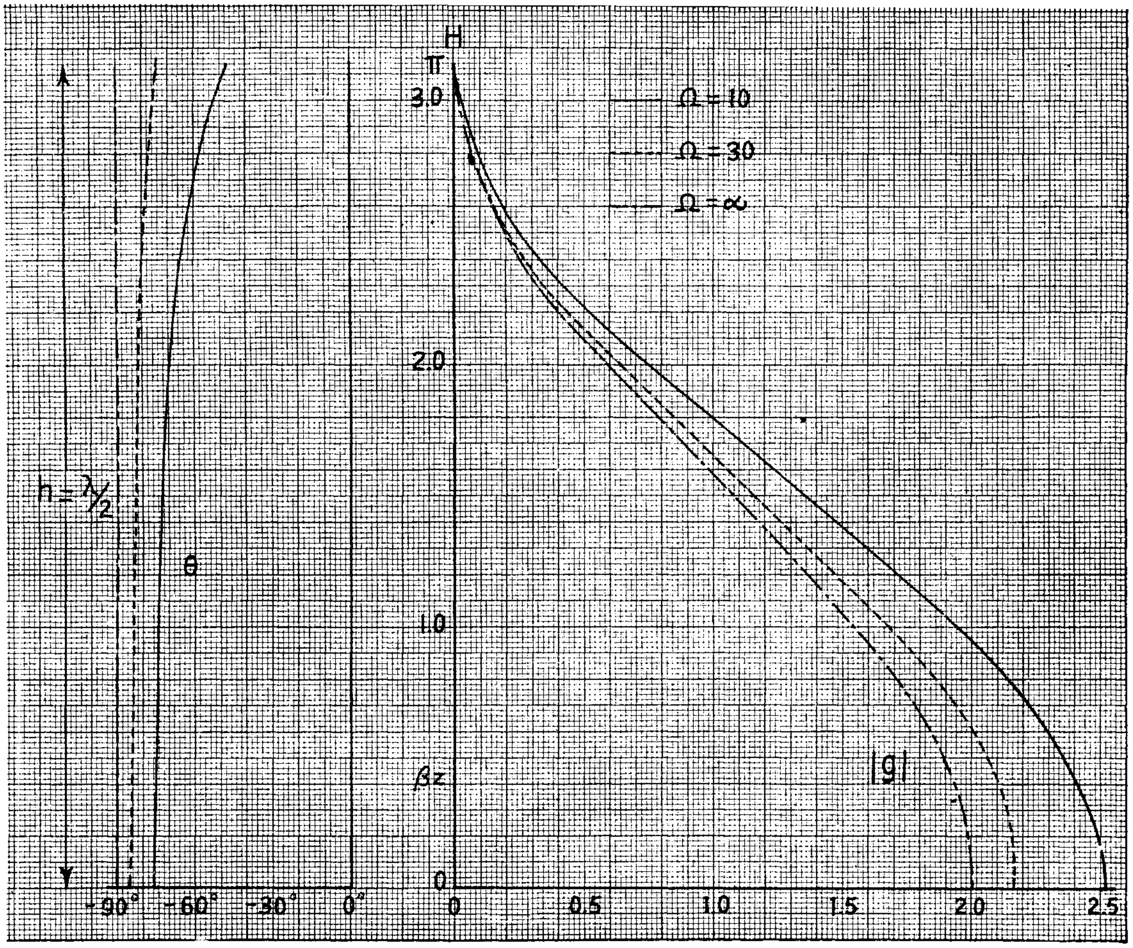

Fig. 9—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $h=\lambda/2$.

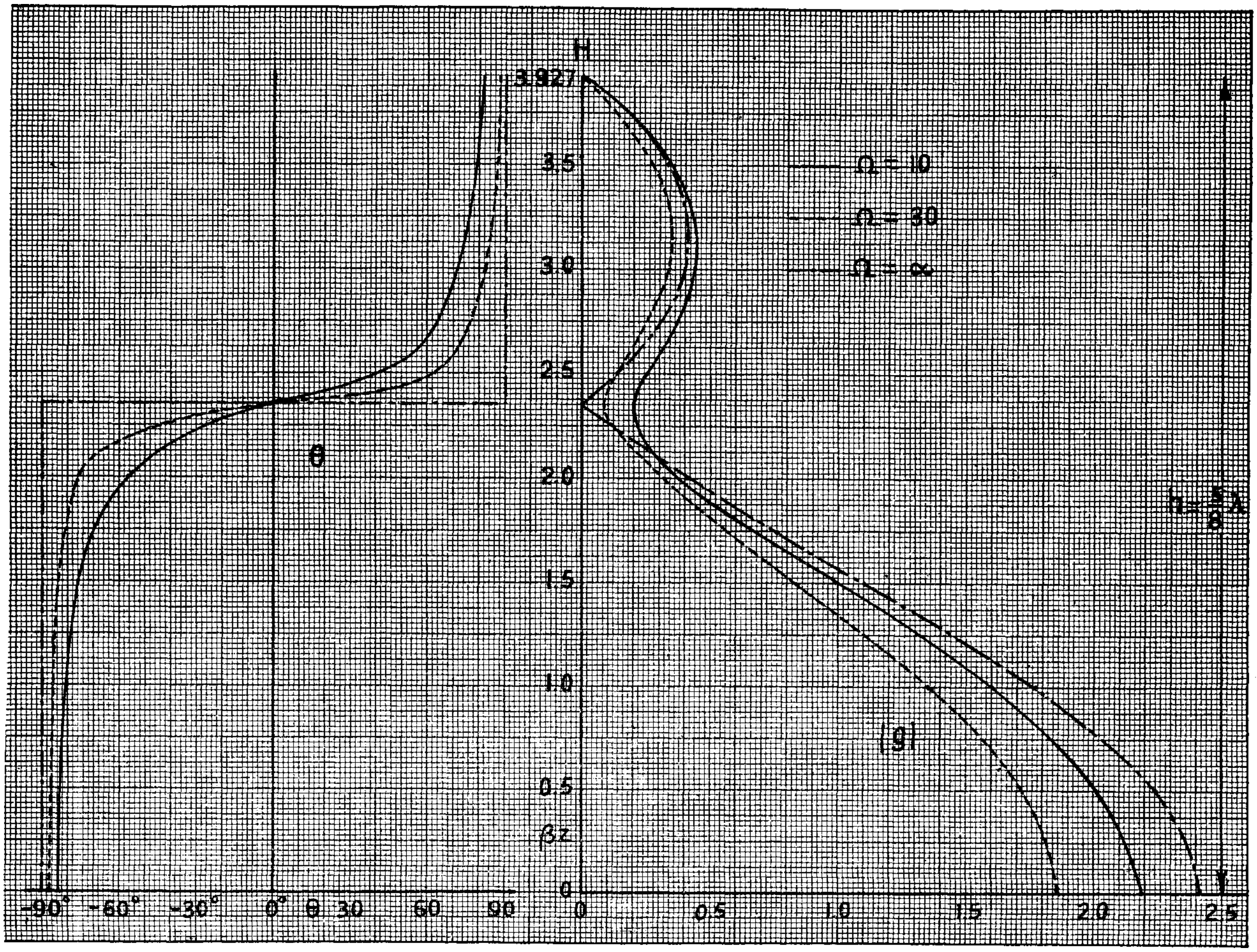

Fig. 10—The amplitude function $|g|$ and the phase angle θ for an unloaded receiving antenna of half-length $h=5\lambda/8$.

of h/a in $\Omega = 2\ln(2h/a)$ is made smaller by increasing the radius a. This may be expressed in terms of a decrease in impedance between two sufficiently closely spaced terminals in the driven antenna. In the receiving antenna an impedance in the conventional sense cannot be defined because the externally applied driving forces are distributed along the entire length of the antenna and not concentrated between two terminals which are well within the near zone with respect to each other. It will be shown below that the receiving antenna can be replaced by an "equivalent" circuit from the point of view of the current at its center in which a fictitious lumped electromotive force is defined. It is important to note that the current in the receiving antenna is directly proportional to the factor

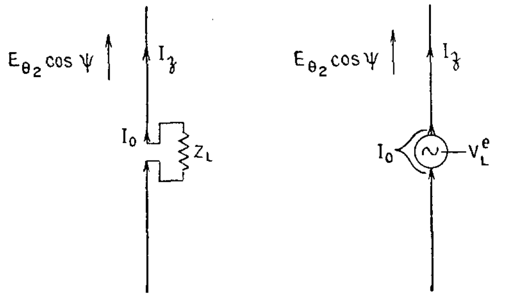

Fig. 11—"Equivalent" circuit for a loaded receiving antenna.

$$U \text{ (volts per wave number)} = (E_{\theta_2} \cos \psi)/\beta = (\lambda E_{\theta_2} \cos \psi)/2\pi. \quad (50)$$

Here $E_{\theta_2} \cos \psi$ is the component of the electric field due to the distant transmitting antenna (number 2) along the axis of the receiving antenna provided this lies in a plane of constant phase of the linearly polarized electric field. The factor U is measured in volts per wave number, where the wave number is the number of wavelengths per meter. The more general case which does not satisfy condition (28) must be treated separately. This has been carried out completely by Hallén[2] only for the limiting case of an infinitely thin antenna with matched load. It is considered in detail in a companion paper.[3]

The Distribution of Current for an Extremely Thin Receiving Antenna

The approximate distribution of current along an extremely thin receiving antenna is obtained from (35) by neglecting the terms in $1/\Omega$ in both numerator and denominator. With (29) it is

$$I_z = (j4\pi U/\Omega R_c)(\cos \beta z - \cos \beta h/\cos \beta h). \quad (51)$$

For the receiving antenna this expression corresponds to (53) in reference 1 for an extremely thin driven antenna. The two formulas are quite different but actually reduce to the same trigonometric form whenever βh is an odd multiple of $\pi/2$. After defining the current I_0 at the center of the receiving antenna by setting $z=0$ in (51) one obtains forms for the receiving antenna corresponding to (55) and (56) in reference 1 for the driven antenna. They are

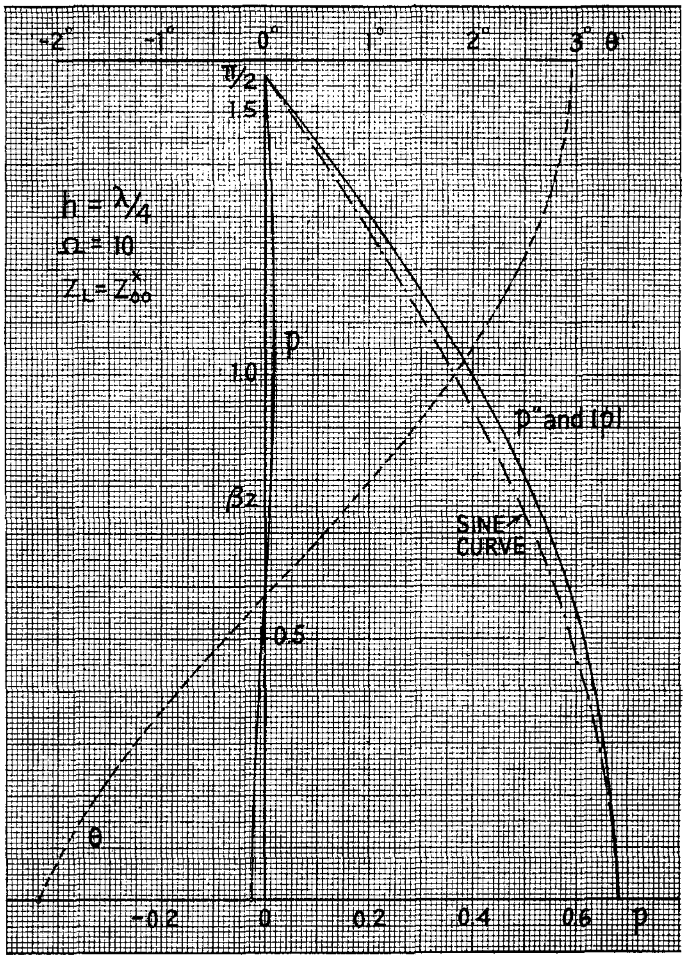

Fig. 12—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=\lambda/4$, $\Omega=10$. p'' is proportional to the component of current in phase with the electric field, p' is proportional to the component in phase quadrature with the field; $|p|$ is proportional to the magnitude of the current; θ is its angle referred to the electric field.

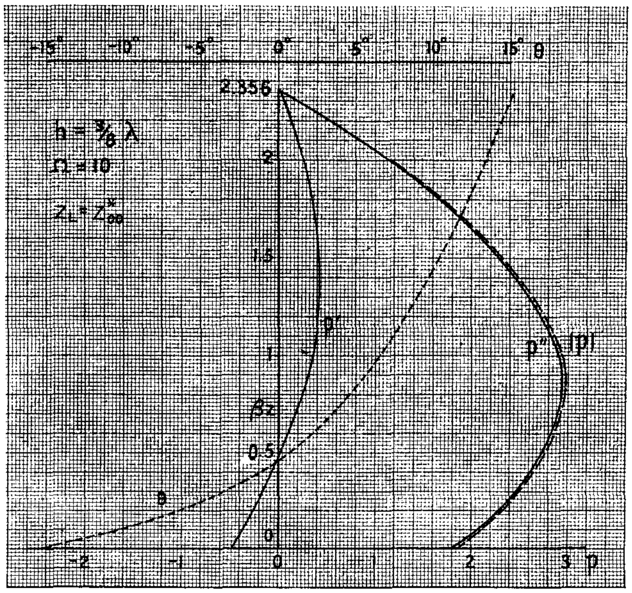

Fig. 13—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=3\lambda/8$; $\Omega=10$.

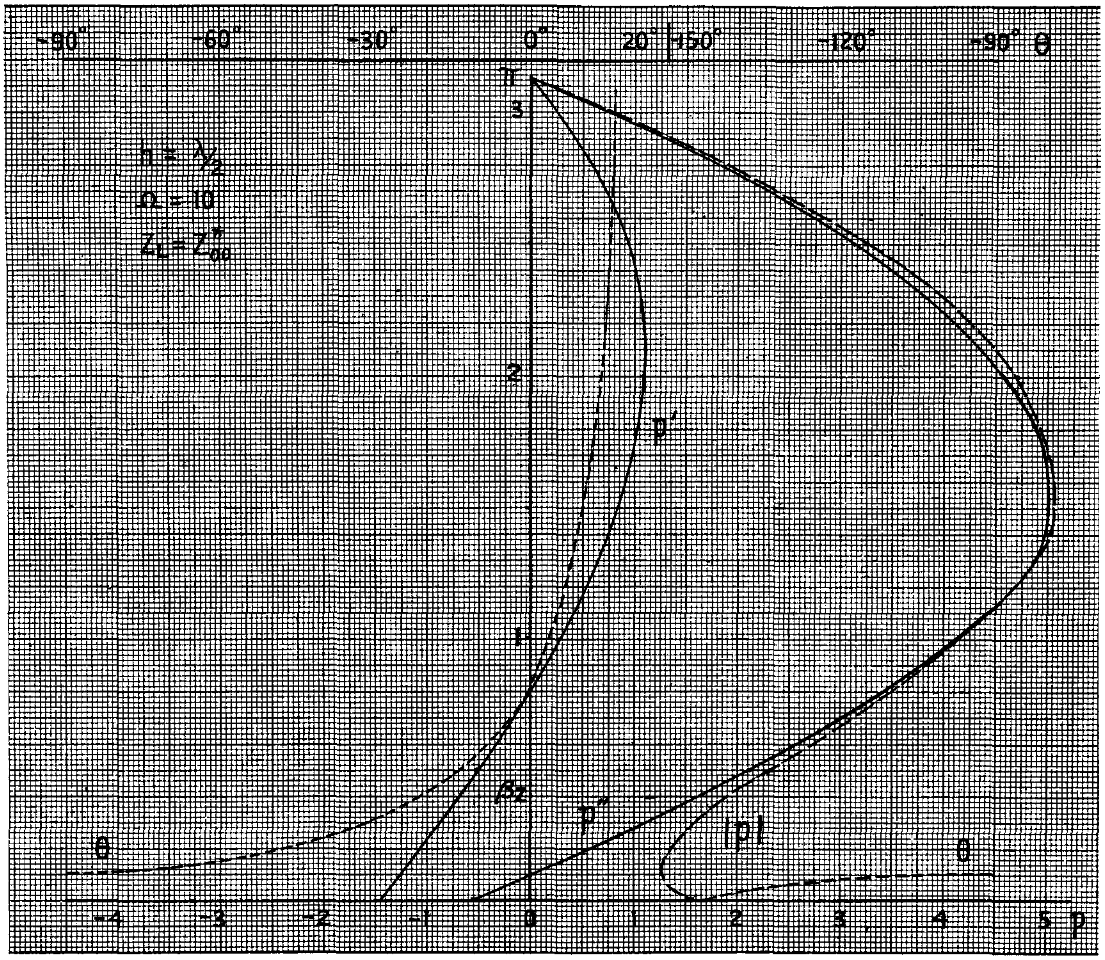

Fig. 14—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=\lambda/2$, $\Omega=10$.

$$I_0 = (j4\pi U/\Omega R_c)(1 - \cos\beta h/\cos\beta h) \qquad (52)$$

$$I_z = I_0(\cos\beta_z - \cos\beta h/1 - \cos\beta h). \qquad (53)$$

The formula (51) is, of course, a good approximation for thicker antennas over the limited ranges of the variables βz and βh for which the trigonometric terms in numerator and denominator of (35) are large compared with the terms in $1/\Omega$. When βh approaches an odd multiple of $\pi/2$, $\cos\beta h$ vanishes. Near such values one can write

$$I_0 = (j4\pi U/\Omega R_c)\{(1 - \cos\beta h)/(\cos\beta h + (1/\Omega)(A_1{}^{I} + A_1{}^{II}))\} \qquad (54)$$

with (53) unchanged. Because the component of current in phase with U is always relatively small compared with the quadrature component except with h near odd multiples of $\lambda/4$ when the distributions of both components are alike, the approximation (53) is a much better one for a receiving antenna than is (56) of reference 1 for a driven antenna. Moreover, one is usually interested in the current at the center of the antenna, and (53) is actually a good approximation for I_z for all values of βh sufficiently below 2π. With $\beta h = 2\pi$, I_z apparently becomes infinite. Actually I_0 simultaneously vanishes and I_z remains finite. Since receiving antennas that satisfy (29) and which have βh much greater than 4 are of no practical importance even when loaded at the center (as will be shown below) (53) with (52), or if required with (54), is a reasonably satisfactory approximation for many purposes.

THE DISTRIBUTION OF CURRENT ALONG A CENTER-LOADED RECEIVING ANTENNA

In normal operation a receiving antenna is connected to a load impedance (which usually consists of a network of tuned circuits) either directly or by means of a transmission line. In some instances it is necessary or desirable to design the load and the antenna so that a maximum of power is transferred from the distant transmitter to the load. In other cases the receiver itself is made sufficiently sensitive so that it need abstract only a negligible amount of power and practically all the power received by the antenna is reradiated. This case then reduces approximately to that of an unloaded antenna.

Consider the general case illustrated in Fig. 11 of a receiving antenna placed in a plane perpendicular to the line joining transmitter and receiver. A load impedance Z_L (which may be the input impedance of a transmission line that is loaded at the output end) is connected across two terminals at its center. These are assumed to be sufficiently close together so that they are in the near zone with respect to each other. With the aid of the compensation theorem the analysis of this circuit can be reduced mathematically to that of an equivalent circuit

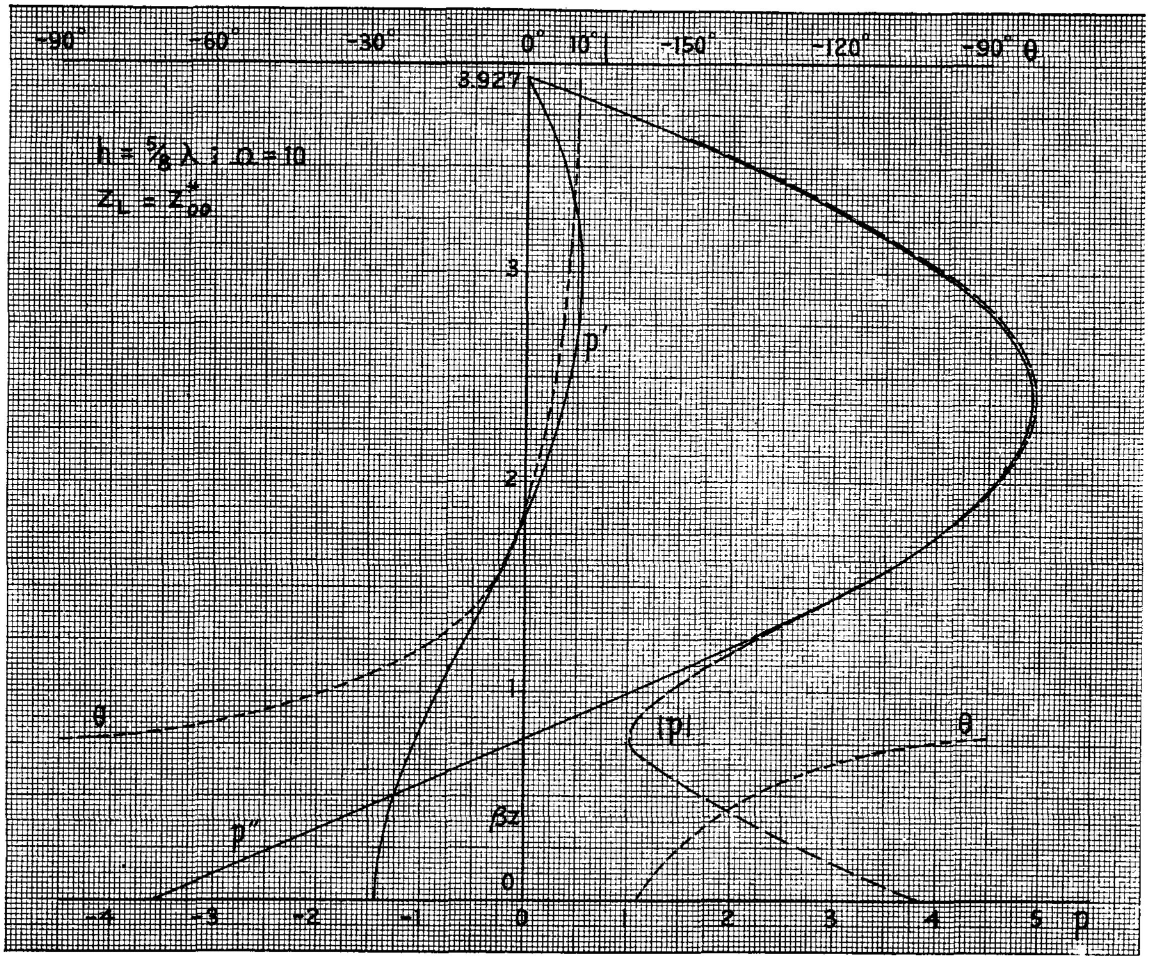

Fig. 15—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=5\lambda/8$, $\Omega=10$.

consisting of an unloaded receiving antenna with an impedanceless generator at its center. According to this theorem any impedance Z_L which carries a current I_0 may be replaced by an impedanceless generator with an impressed potential difference or electromotive force $V_L{}^e$ equal to the potential drop across the impedance. That is, with

$$V_L{}^e = I_0 Z_L. \tag{55}$$

If this substitution is made the circuit on the right in Fig. 11 is equivalent to that on the left.

By direct application of the principle of superposition the total current I_0 is the algebraic sum of the current I_{0V} due to the generator, and the current I_{0E} due to the action of the distant transmitter as expressed in the electric field E thus,

$$I_0 = I_{0E} - I_{0V}. \tag{56}$$

The current I_{0V}, due to the impedanceless generator, is the same as that in a driven antenna. Thus,

$$I_{0V} = (V_L{}^e/Z_{00}) = I_0(Z_L/Z_{00}). \tag{57}$$

Here Z_{00} is the input self-impedance of a center-driven antenna which has the same dimensions as the receiving antenna under consideration.[4] With (55) and (57), (56) may be written as follows:

$$\begin{aligned} I_{0E} &= I_0 + I_{0V} = V_L{}^e((1/Z_L) + (1/Z_{00})) \\ &= I_0(1 + (Z_L/Z_{00})). \end{aligned} \tag{58}$$

At any point z along the antenna the current I_z is by the principle of superposition also the algebraic sum of the separate currents due to the electric field and due to the generator. Thus,

$$I_z = I_{zE} - I_{zV}. \tag{59}$$

With (46) and (43) in reference 1 this may be expanded as follows:

$$\begin{aligned} I_z = {} & (1/30\Omega D)\{U[g''(z) + jg'(z)] \\ & - \tfrac{1}{2}V_L{}^e[f''(z) + jf'(z)]\}. \end{aligned} \tag{60}$$

With (58) and setting $z=0$ in (46), (55) becomes

$$\begin{aligned} V_L{}^e &= I_{0E}(Z_{00}Z_L/Z_{00} + Z_L) \\ &= (U/30\Omega D)[g''(0) + jg'(0)](Z_{00}Z_L/Z_{00} + Z_L). \end{aligned} \tag{61}$$

Upon substituting (61) in (60) one obtains the following general equation for the current in the loaded receiving antenna in terms of the real factors p'' and p' which are, respectively, proportional to the components of current in phase and in phase quadrature with U.

$$I_z = (U/30\Omega D)(p'' + jp'). \tag{62}$$

Here $U = (E_{\theta_2} \cos\psi)/\beta = (\lambda E_{\theta_2} \cos\psi)/2\pi$ (63)

$$\begin{aligned} p'' + jp' = {} & [g''(z) + jg'(z)] - (1/60\Omega D) \\ & (Z_{00}Z_L/Z_{00} + Z_L)(g''(0) + jg'(0)) \cdot [f''(z) + jf'(z)]. \end{aligned} \tag{64a}$$

The notation, $|p| = \sqrt{(p')^2 + (p'')^2}$;

$$\theta = \tan^{-1}(p'/p'') \tag{64b}$$

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[4] Ronold King and F. G. Blake, Jr., "The self-impedance of a symmetrical antenna," Proc. I.R.E., vol. 30, pp. 335–349; July, 1942.

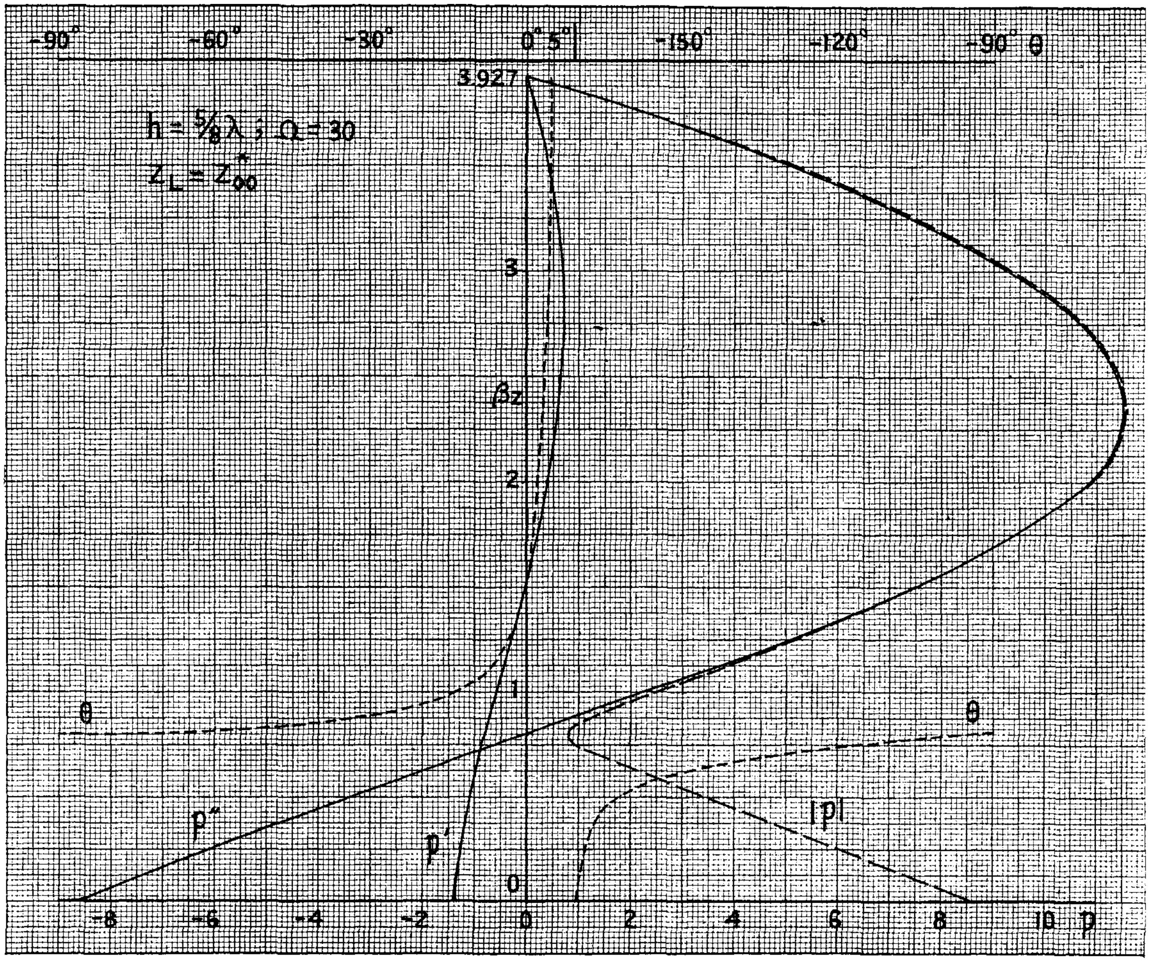

Fig. 16—The functions p', p'', $|p|$, and θ for a receiving antenna with a conjugate-matched load. $h=5\lambda/8$, $\Omega=30$.

will be used. Since Z_{00} contains 60Ω as a factor, the second set of terms in (64a) is not negligible unless Z_L becomes sufficiently small. If the antenna is sufficiently thin (62) with (64a) becomes

$$\begin{aligned} I_z = {} & (U/30\Omega \cos\beta h)\{j(\cos\beta z - \cos\beta h) \\ & + (1/60\Omega\cos\beta h)(Z_{00}Z_L/Z_{00}+Z_L) \\ & \cdot(1-\cos\beta h)\sin\beta(h-|z|)\}. \end{aligned} \tag{65}$$

The distributions of current along a center-loaded receiving antenna for several different loads, lengths, and thicknesses are shown in Figs. 12 to 18 in terms of the functions p', p'', $|p|$, and θ. They were calculated from the general formula (62) using the distribution functions g' and g'' given in Figs. 3 to 6 and the distribution functions f' and f'' shown in reference 1, Figs. 7 to 10.

If the load is conjugate-matched to the antenna so that $Z_L = Z_{00}^*$ one has

$$\begin{aligned} (Z_{00}Z_L/Z_{00}+Z_L) &= (Z_{00}Z_{00}^*/Z_{00}+Z_{00}^*) \\ &= (R_{00}^2 + X_{00}^2/2R_{00}) = (1/2G_{00}). \end{aligned} \tag{66}$$

Figs. 12 to 16 were calculated using (66) in (64a). In Fig. 17 the load consisted of a reactance such that $X_L = -X_{00}$ while the resistance was made very much greater than the self-resistance R_{00} of the antenna. In Fig. 18, $X_L = -X_{00}$ while $R_L = 0$. Very considerable differences in the distribution of current are seen to exist in antennas of the same length but with different impedances at the center. In particular, whenever the antenna and its load are made self-resonant the distribution of current resembles that along a center-driven antenna of the same length much more than that along an unloaded receiving antenna. This effect is increased if the load resistance is made as small as possible, as can be seen very readily from (64a). Clearly if the factor $(Z_{00}Z_L/Z_{00}+Z_L)(g''(0)+jg'(0))$ in (64a) is made sufficiently large and predominantly real, the distribution function $f''(z)+jf'(z)$ may be made to predominate. This is precisely the distribution function for a center-driven antenna. It may be approximated by requiring $R_L=0$; $X_L=-X_{00}$, i.e., by tuning the antenna to resonance with a pure reactance connected across its terminals. The distribution of current for these conditions is shown in Fig. 17 for one length and radius. The importance of this fact for coupled antennas will be considered in a subsequent paper.

Since the functions g' and g'', and p' and p'' are plotted in Figs. 3 to 18, a direct comparison of the amplitude of current for antennas of different thickness is possible only if these functions are multiplied by the factor $U/30\Omega D = \lambda E_{\theta_2}\cos\psi/60\pi\Omega D$. If the antennas (the currents in which are to be compared) are similarly oriented in the same electric field at the same frequency

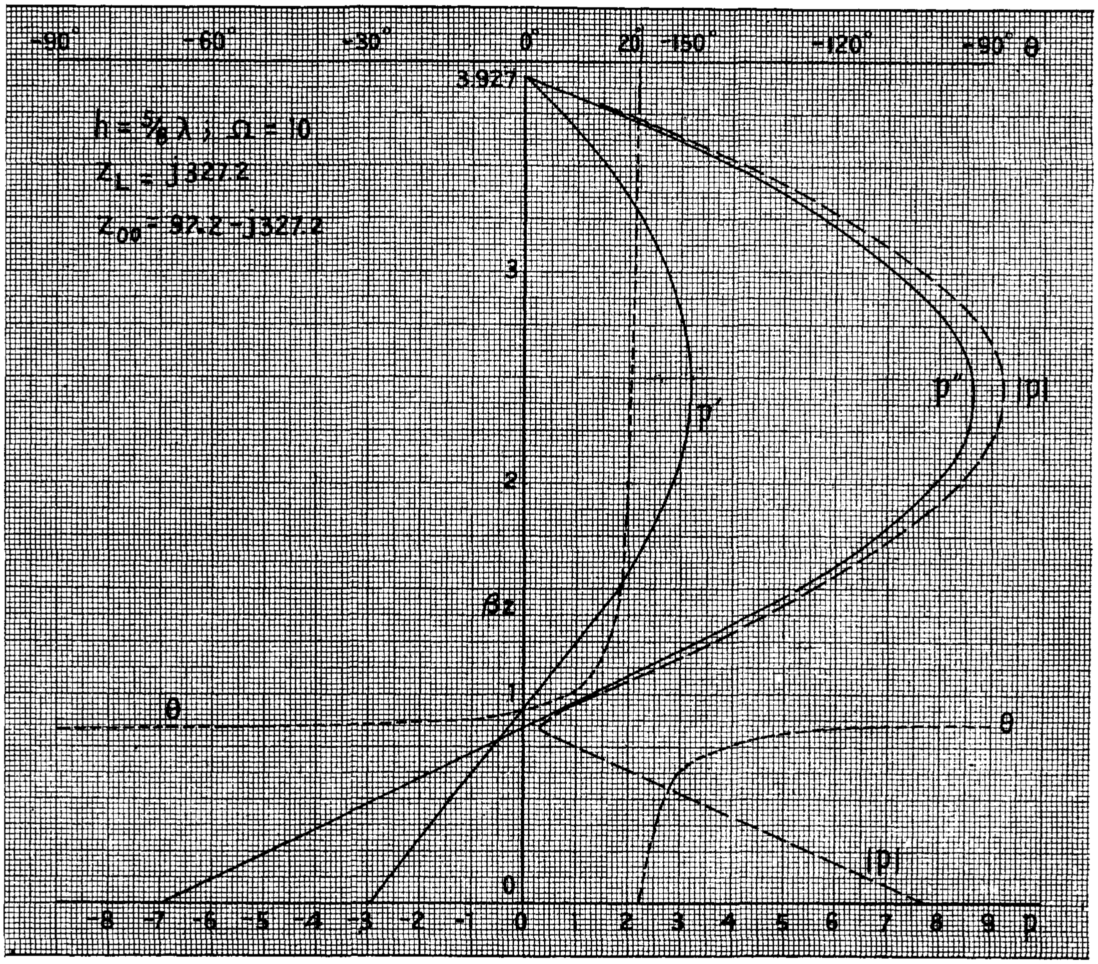

Fig. 17—The functions p', p'', $|p|$, and θ for a receiving antenna with a purely reactive load which is the negative of the input reactance of the antenna. $h=5\lambda/8$, $\Omega=10$.

multiplication by the factor $1/60\pi\Omega D$ is sufficient. This factor is given in Table I for antennas of two thicknesses and four lengths. In Table II are shown the comparable magnitudes of the currents at the centers of unloaded receiving antennas of different lengths in the form $|I_0|/\lambda E_{\theta_2} \cos\psi$. It is clear that the amplitude of current is large near self-resonance and small near antiresonance. Precisely self-resonant and antiresonant lengths are not tabulated. In Fig. 19 the magnitude $|I_z/\lambda E_{\theta_2}\cos\psi|$, is plotted for all antennas of half-length $h=(5/8)\lambda$ previously computed and separately plotted in order to make ready intercomparison of both magnitudes and distributions of current possible under different load conditions and for different thicknesses but when immersed in exactly the same field.

TABLE II

h/λ	$\Omega = 2\ln\dfrac{2h}{a}$	$\lvert I_0\rvert/\lambda E_{\theta_1}\cos\psi$
0.25	10	9.35×10^{-3}
	30	8.10
0.375	10	3.08
	30	0.92
0.5	10	2.92
	30	0.82
0.625	10	3.91
	30	0.98

The Equivalent Circuit and the Effective Length of a Loaded Receiving Antenna Oriented to Lie in an Equiphase Plane of a Linearly Polarized Electric Field

The relation (58) may be rearranged as follows:

$$I_{0E}Z_{00} = I_0(Z_{00} + Z_L). \tag{67}$$

This equation gives the correct amplitudes of the current I_0 entering or leaving the load Z_L connected at the center of a symmetrical receiving antenna that is placed in the plane perpendicular to the line joining it to a distant transmitter. Actually, however, it is also the equation of the same antenna with the same load at its center but driven by an impedanceless generator as shown in Fig. 20. The electromotive force of the generator is

$$V_0^e = I_{0E}Z_{00}. \tag{68}$$

Thus the forces acting on the charges distributed all along the receiving antenna in the actual circuit are replaced in the "equivalent" circuit by forces due to a generator acting as a concentrated electromotive force at the center. For the purpose of determining the current I_0 the two circuits are equivalent; but not for all purposes. For example, the distribution of current along the receiving antenna is given by (62), whereas the distribution along the antenna in the equivalent circuit is

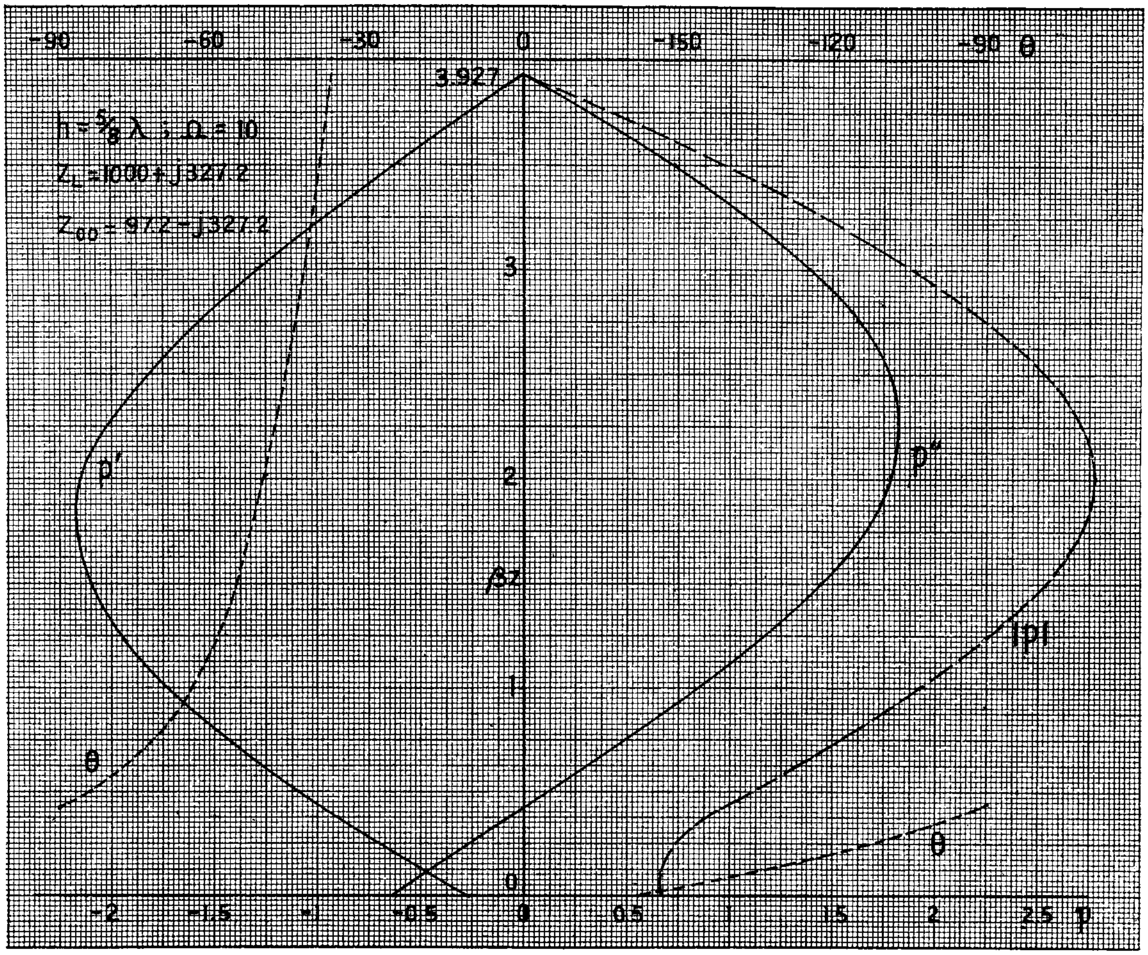

Fig. 18—The functions p', p'', $|p|$, and θ for a receiving antenna with a load including a resistance of 1000 ohms and a reactance which is the negative of the input reactance of the antenna. $h=5\lambda/8$, $\Omega=10$.

given by (43) of reference 1. The two distributions are not at all similar and they lead to quite different electromagnetic fields. That is, the power which would be radiated from the "equivalent" driven antenna would be transferred to different parts of the distant universe than is the power reradiated from the actual antenna. Whereas the fictitious electromotive force (68) is so chosen that the current I_0 is the same in the actual and the "equivalent" circuits, the total power radiated from the driven antenna in the "equivalent" circuit is not equal to the power actually reradiated from the receiving antenna. From the point of view of the power absorbed in the load Z_L, this is immaterial and the "equivalence" of the two circuits is entirely adequate.

The electromotive force $V_0{}^e$ of the impedanceless generator in the "equivalent" circuit as defined by (68) is readily expressed analytically. Thus, with $z=0$ in (46),

$$V_0{}^e = (U/30\Omega D)[g''(0) + jg'(0)]Z_{00}. \qquad (69)$$

It may be expressed directly in terms of the electric field using (29). Thus, defining a complex "effective" length, $2(h_e'' + jh_e') = 2h_e e^{j\phi}$, for the antenna of actual length $2h$ one can write

$$V_0{}^e = 2|h_e| E_{\theta_2} \cos\psi \cdot e^{j\phi} \qquad (70)$$

$$\text{with} \quad h_e'' + jh_e' = |h_e| e^{j\phi} = \frac{[g''(0) + jg'(0)]Z_{00}}{60\beta\Omega D}. \qquad (71)$$

Here Z_{00} is the self-impedance of the antenna as seen by a generator connected in place of the load. In most cases involving a linearly polarized electric field one is not at all interested in the relative phase relations between the electromotive force in the "equivalent" series circuit and the electric field parallel to the antenna. Accordingly the angle ϕ is of little or no consequence and it is adequate to compute as the real "effective" half-length of the antenna the magnitude $|h_e|$ of (71). It is

$$|h_e| = (|g(0)||Z_{00}|/60\beta\Omega D). \qquad (72a)$$

The "effective" electrical length is

$$H_e = \beta h_e = (2\pi h_e/\lambda). \qquad (72b)$$

The electromotive force in the equivalent series circuit then has the magnitude

$$|V_0{}^e| = 2|h_e||E_{\theta_2}| \cos\psi. \qquad (73)$$

It is easily verified by the application of Thévenin's theorem at the terminals of the antenna to which the load is attached that (70) must be the open-circuit voltage across these terminals. The effective length of a center-loaded, isolated antenna immersed in a plane-polarized electric field may be defined in general to be the open-circuit voltage at the load terminals of the antenna divided by the component of the electric field in the plane containing the antenna and the wave normal (a line perpendicular to a wave front or surface of

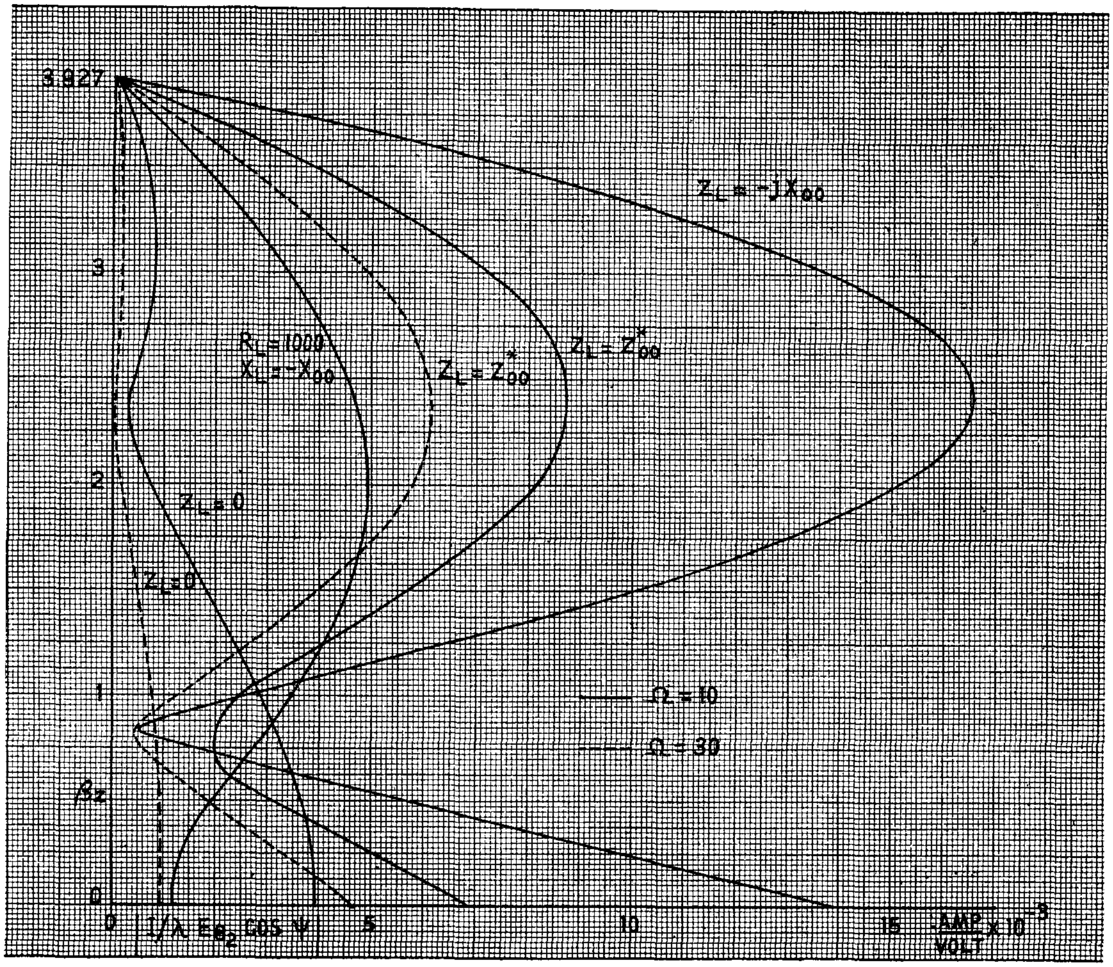

Fig. 19—The amplitudes $|(I/\lambda E_{\theta_2}\cos\psi)|$ for the antennas of Figs. 10 and 15 to 18. This figure permits comparison of the actual amplitudes of current in antennas of the same length immersed in the same electric field at the same frequency. $h=5\lambda/8$.

constant phase). This component is $E_{\theta_2}\cos\psi$ in the notation of the present paper. It is the component parallel to the antenna *only in the special case defined by* (28), i.e., when the antenna is itself perpendicular to the wave normal so that it lies in a plane of constant phase. For an indefinitely thin antenna using (54) of reference 1 this reduces to the real form given below.[5,6]

$$h_e = (1 - \cos\beta h/\beta \sin\beta h) = (\lambda/2\pi)\tan(\tfrac{1}{2}\beta h). \quad (74)$$

Formulas (72a) and (74) are plotted in Fig. 21. It is important to note that the simple form (74) is a good approximation even for thick antennas for lengths sufficiently below $h=\lambda/2$. The maxima of (72a) occur at the antiresonant lengths as obtained from (29b) of reference 4. Using the "effective" half-length given by (72) or where the approximation is adequate, by (74), the electromotive force in the "equivalent" series circuit of the receiving antenna is readily determined from (70) or (73) if the component of the electric field parallel to the antenna is known either from computation or by measurement and the antenna lies in an equiphase surface. It is important to point out again that the above

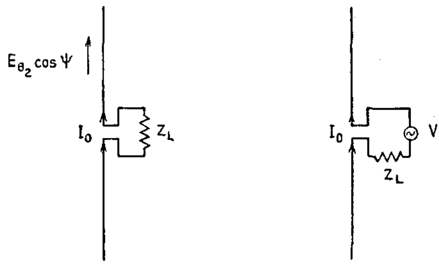

Fig. 20—"Equivalent" circuit of a loaded receiving antenna for determining current in the load.

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[5] The effective half-length or height of a symmetrical antenna (or the effective full length or height of a vertical antenna erected over a perfect conductor) has been defined to be $\int I_z dz$ divided by I_0, where I_z is the total vertical current in the antenna structure at height z and I_0 is the current at the point where power is transferred to or from the connected apparatus. This definition of "effective height" for a receiving antenna leads approximately to the simple special form (74) for an indefinitely thin antenna only in the special case of a load for which $X_L=-X_{00}$. Only in this case is the simple distribution function $I_z=I_0 \sin\beta(h-|z|)$ a moderately good approximation in a receiving antenna, and it is this function which, when substituted in $(\int I_z dz)/I_0$, gives (74). It has already been pointed out[6] that it is possible to define an effective height for a receiving antenna in terms of $(\int I_z dz)/I_0$ using the correct distribution of current for an unloaded receiving antenna. But the effective height so defined does not, when multiplied by the component of the electric field parallel to the antenna, give the electromotive force of a fictitious generator in series with a simple circuit consisting of the load and the input impedance of the antenna as a driven antenna, as does that defined by (71) (or approximately over a limited range that defined by (74) in the special case defined by (28).

[6] Ronold King, "The approximate representation of the distant field of linear radiators," PROC. I.R.E., vol. 29, p. 461; August, 1941.

relations are valid *only if the receiving antenna is* (1) *in the radiation zone of the transmitter and* (2) *in the plane perpendicular to the line joining receiving and transmitting antennas* or more generally, perpendicular to the wave normal. The effective length of an antenna which satisfies (1) but not (2) is considered in a companion paper.[3]

MAXIMUM TRANSFER OF POWER TO THE LOAD

The condition of maximum transfer of power to the load in the "equivalent" series circuit is that of a conjugate match as defined by $Z_L = Z_{00}^*$. In this case the total power supplied to the load is given by

$$P_L = (|V_0^e|^2/4R_{00}) = (|E_{\theta_2}h_e|^2 \cos^2\psi/R_{00}). \tag{75}$$

Here V_0^e and E_{θ_2} are root-mean-square values. With (72) this reduces to

$$P_L = (E_{\theta_2}\cos\psi/60\beta\Omega D)^2(|g(0)|^2/G_{00});$$
$$|g(0)| = \sqrt{[g'(0)]^2 + [g''(0)]^2}. \tag{76}$$

This may be written in the form

$$P_L = ((E_{\theta_2}\lambda\cos\psi)^2/240\pi^2)f(H) \tag{77a}$$

with

$$f(H) = (60|g(0)|/(60\Omega D)^2 G_{00}). \tag{77b}$$

For an indefinitely thin antenna, for which (74) may be substituted in (75), one can also write

$$R_{00} = R_m^e/\sin^2 H. \tag{78}$$

Here R_m^e is the radiation resistance referred to maximum sinusoidal current as plotted in Fig. 21[1] in the curve marked $\Omega = \infty$. The abbreviation $H = \beta h$ is used. With (74) and (78) used in (75),

$$P_L = (E_{\theta_2}\cos\psi/\beta)^2 \cdot ((1 - \cos H)^2/R_m^e). \tag{79}$$

This may be written in the alternative form

$$P_L = ((E_{\theta_2}\lambda\cos\psi)^2/240\pi^2)f(H) \tag{80a}$$

with

$$f(H) = (60(1 - \cos H)^2/R_m^e). \tag{80b}$$

This is equivalent to the result given by Hallén's,[2] formula (45) for the special case defined by (28). Curves

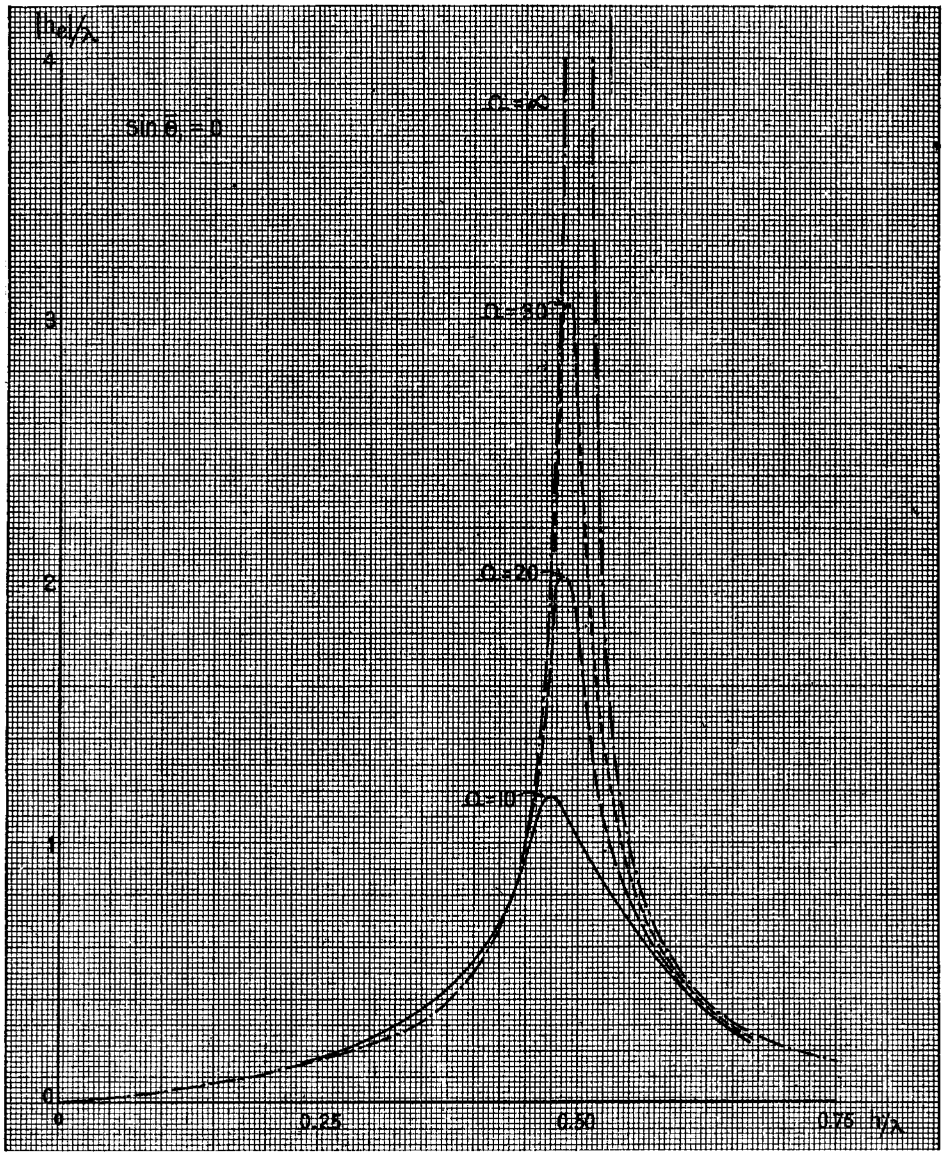

Fig. 21—The effective half-length $|h_e|$ of center-loaded cylindrical antennas of half-length h, or the effective length $|h_e|$ of base-loaded cylindrical antennas erected vertically over a perfectly conducting half space.

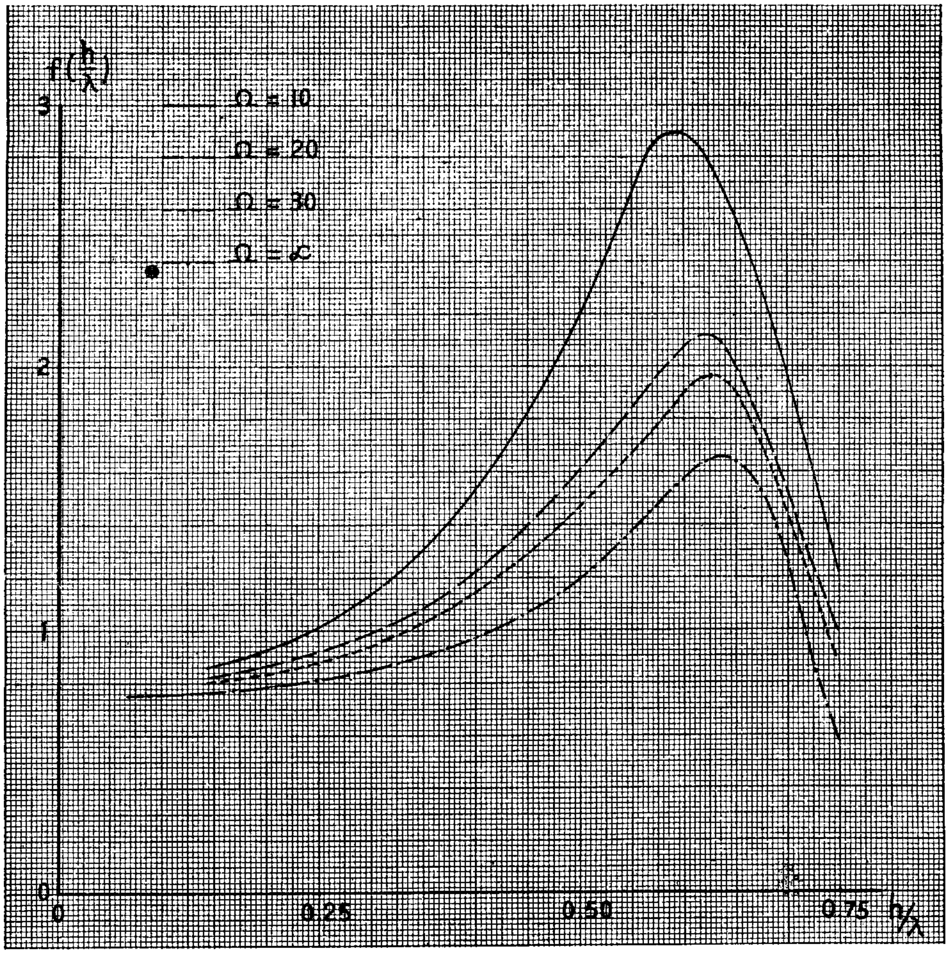

Fig. 22—The function $f(h/\lambda)$ which is proportional to the power transferred to a conjugate-matched load in a center-loaded receiving antenna.

of $f(H)$ as a function of H as computed from (77b) for three thicknesses as defined by $\Omega = 10, 20, 30$, are shown in Fig. 22, together with the curve computed from (80b) and defined by $\Omega = \infty$. It is seen that the *optimum length for transferring a maximum power to a matched load is somewhat below the half length $h = (5/8)\lambda$ and more than twice the self-resonant half-length near $h = \lambda/4$, which gives maximum currents with no load.* Moreover the power transferred to the matched load is greater for thick than for thin antennas.

APPENDIX

$$\begin{aligned}
S_1(z) = &-(\cos qz - \cos qh) \ln (1 - z^2/h^2) \\
&+ \tfrac{1}{2} \cos qz \big[\overline{Ci}(\beta+q)(h+z) + \overline{Ci}(\beta-q)(h-z) \\
&+ \overline{Ci}(\beta-q)(h+z) + \overline{Ci}(\beta+q)(h-z) \\
&+ jSi(\beta+q)(h+z) + jSi(\beta-q)(h-z) \\
&+ jSi(\beta+q)(h-z) + jSi(\beta-q)(h+z)\big] \\
&- \tfrac{1}{2} \sin qz \big[Si(\beta+q)(h+z) + Si(\beta-q)(h-z) \\
&- Si(\beta-q)(h+z) - Si(\beta+q)(h-z) \\
&- j\overline{Ci}(\beta+q)(h+z) - j\overline{Ci}(\beta-q)(h-z) \\
&+ j\overline{Ci}(\beta-q)(h+z) + j\overline{Ci}(\beta+q)(h-z)\big] \\
&- \cos qh \big[\overline{Ci}\beta(h+z) + \overline{Ci}\beta(h-z) \\
&+ jSi\beta(h+z) + jSi\beta(h-z)\big]
\end{aligned}$$

$$\begin{aligned}
&+ \frac{j4\pi z^i h}{R_c} \left[\frac{\beta(\cos qz - \cos \beta z)}{h(\beta^2 - q^2)} \right. \\
&\left. - \frac{\cos qh}{\beta h} (1 - \cos \beta z)\right] \cdot
\end{aligned} \tag{81}$$

$$S_1(z) = 0 \text{ for } q = 0 \tag{82}$$

$$\begin{aligned}
S_1(h) = &\tfrac{1}{2} \cos qh \big[\overline{Ci}2h(\beta+q) + \overline{Ci}2h(\beta-q) + jSi2h(\beta+q) \\
&+ jSi2h(\beta-q)\big] - \tfrac{1}{2} \sin qh \big[Si2h(\beta+q) - Si2h(\beta-q) \\
&- j\overline{Ci}2h(\beta+q) + j\overline{Ci}2h(\beta-q)\big] \\
&- \cos qh \big[\overline{Ci}2\beta h + jSi2\beta h\big] \\
&+ j\frac{4\pi z^i h}{R_c} \left[\frac{\beta(\cos qh - \cos \beta h)}{h(\beta^2 - q^2)} \right. \\
&\left. - \frac{\cos qh}{\beta h} (1 - \cos \beta h)\right] \cdot
\end{aligned} \tag{83}$$

$$S_1(h) = 0 \text{ for } q = 0. \tag{84}$$

$$Six = \int_0^x \frac{\sin u}{u} \, du \tag{85}$$

$$\overline{Ci}x = \int_0^x \frac{1 - \cos u}{u} \, du \tag{86}$$

z^i is the internal impedance per unit length of the antenna.